НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ ЦЕНТР
«КУРЧАТОВСКИЙ ИНСТИТУТ»

ПЕТЕРБУРГСКИЙ ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
Россия, 188300, Ленинградская область, г. Гатчина, Орлова роща

IV Школа по физике поляризованных нейтронов

Б. П. Топерверг (НИЦ КИ ПИЯФ, Гатчина

Кинематика неупругого рассеяния поляризованных нейтронов (при малых углах скольжения к поверхности?)

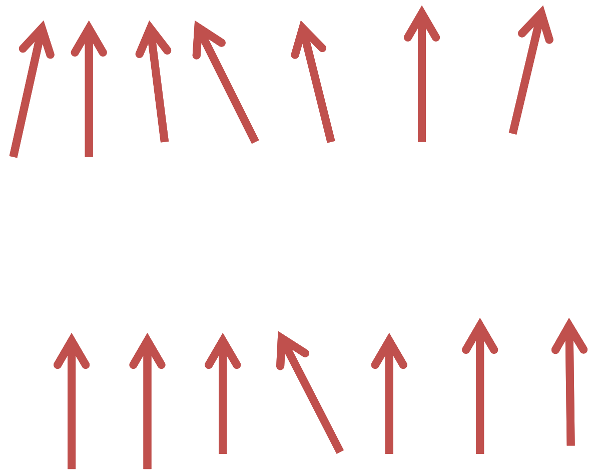
Spin Dynamics in ideal Heisenberg ferromagnet

Mean field

Hamiltonian:

$$\mathrm{H} = -\sum_{l}\left[\sum_{l\neq l'} J_{ll'}(\vec{S}_l \cdot \vec{S}_{l'}) + \mu\vec{H}\cdot\vec{S}_l\right] = \mathrm{H}_0 + \mathrm{H}_{\mathrm{int}}$$

$$\vec{S}_l = \left\langle \vec{S}_l \right\rangle + \Delta\vec{S}_l, \qquad \Delta\vec{S}_l = \left(\vec{S}_l - \left\langle \vec{S}_l \right\rangle\right)$$



$$\left\langle \vec{S}_j \right\rangle = \mathrm{Tr}\{e^{-\mathbf{H}/T}\vec{S}_j\} \text{ is the mean value}$$

$$\mathrm{H}_0 \approx -\mu\sum_{l}(\vec{H}_l \cdot \vec{S}_l), \text{ where}$$

$$\mu\vec{H}_l = \sum_{l'} J_{ll'}\left\langle \vec{S}_{l'} \right\rangle + \mu\vec{H} \text{ is mean field,}$$

$$\mathrm{H}_{\mathrm{int}} = -\frac{1}{2}\sum_{l} J_{ll'}\left(\vec{S}_l - \left\langle \vec{S}_l \right\rangle\right)\cdot\left(\vec{S}_{l'} - \left\langle \vec{S}_{l'} \right\rangle\right)$$

Homogeneous Spin Precession in Mean Field

$\dot{\vec{S}}_l(t) = \mu[\vec{S}_l(t) \times \vec{H}_l]$, classical equation of motion

$\vec{S}_l(t) = \hat{R}(t)\vec{S}_l(0)$, where precession matrix is

$R_l^{\alpha\beta}(t) = h_l^\alpha h_l^\beta + (\delta^{\alpha\beta} - h_l^\alpha h_l^\beta)\cos(\Omega_l t) + \varepsilon^{\alpha\beta\gamma} h_l^\gamma \sin(\Omega_l t)$,

$\Omega_l = \mu H_l/\hbar$ precession frequency, $\vec{h} = \vec{H}_l/H_l$ and $H_l = |\vec{H}_l|$

$\mu H_l \sim J \sim 1000\mathrm{K} \approx 86\,\mathrm{meV}$, $\Omega_l/2\pi \sim 20\,\mathrm{THz}$

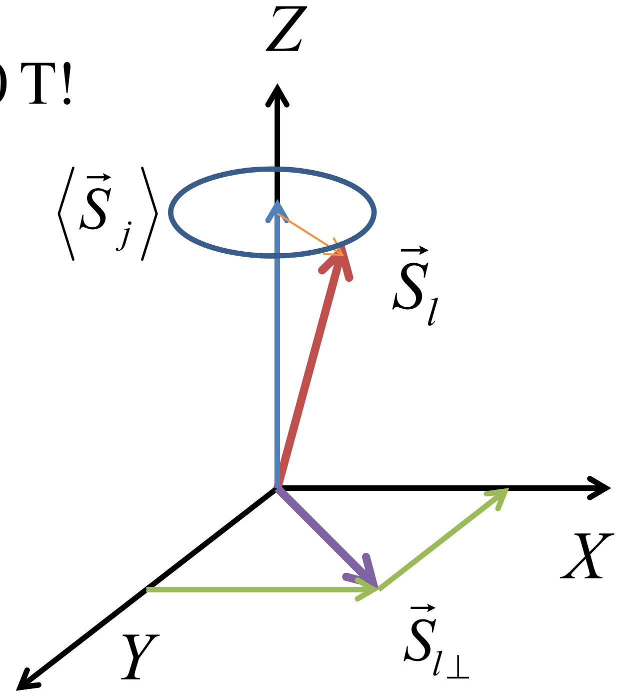
$H_l \sim J/\mu$ $\mu_\mathrm{B} = 0.058\,\mathrm{meV}\cdot\mathrm{T}^{-1}$, $H_l \sim 700\,\mathrm{T}$!

In particular coordinate system

$S_l^z(t) = S_l^z(0)$,

$S_l^x(t) = S_l^\perp \cos(\Omega_l t + \phi)$

$S_l^y(t) = S_l^\perp \sin(\Omega_l t + \phi)$



Precession: Single-site autocorrelations

Single - site spin correlation function :

$$\left\langle S_l^\alpha(t) S_l^\beta(t')\right\rangle = \mathrm{Tr}\{e^{-\mathbf{H}/T} S_l^\alpha(t) S_l^\beta(t')\}$$ is the thermodynamic mean value

Spin correlator :

$$K^{\alpha\beta}(\omega) = \left\langle S_l^\alpha(\omega) S_l^\beta(-\omega)\right\rangle = \frac{1}{t}\int_{-t/2}^{t/2} dt'\,dt''\,e^{-i\omega(t'-t'')}\left\langle S_l^\alpha(t') S_l^\beta(t'')\right\rangle$$

$$S_l^\alpha(t) = e^{i\mathbf{H}t} S_l^\alpha e^{-i\mathbf{H}t}$$

Tensor structure : $$\begin{pmatrix} K_{\perp s}(\omega) & -iK_{\perp a}(\omega) & 0 \\ iK_{\perp a}(\omega) & K_{\perp s}(\omega) & 0 \\ 0 & 0 & K_{\|}(\omega) \end{pmatrix}$$

Transverse symmetric: $$K_{\perp s}(\vec{q},\omega) = \frac{\pi T}{\hbar\Omega}\{\delta(\omega-\Omega)+\delta(\omega+\Omega)\},$$

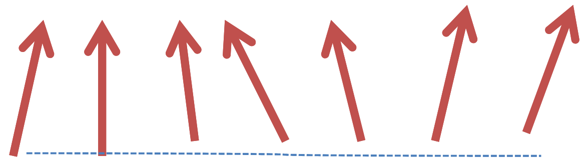
Transverse antisymmatric : $$K_{\perp a}(\vec{q},\omega) = \frac{\pi T}{\hbar\Omega}\{\delta(\omega-\Omega)-\delta(\omega+\Omega)\},$$

Longitudinal correlator : $$K_{\|}(\omega) = 0$$

Spin-spin correlations: coupled precession

Pair spin correlation function:

$$\left\langle S_l^\alpha(t) S_{l'}^\beta(t')\right\rangle = \mathrm{Tr}\{e^{-\mathbf{H}/T} S_i^\alpha(t) S_j^\beta(t')\}$$



Spin correlator:

$$K^{\alpha\beta}(\vec{q},\omega) = \left\langle S_{\vec{q}}^\alpha(\omega) S_{-\vec{q}}^\beta(-\omega)\right\rangle = \frac{1}{N}\sum_{ll'} e^{i\vec{q}(\vec{r}_l - \vec{r}_{l'})}\left\langle S_l^\alpha(\omega) S_{l'}^\beta(-\omega)\right\rangle$$

Tensor structure: $K^{\alpha\beta}(\vec{q},\omega) = (\delta^{\varepsilon\beta} - h^\alpha h^\beta) K_{\perp s} - i\varepsilon^{\alpha\beta\gamma} h^\gamma K_{\perp a} + h^\alpha h^\beta K_{\|}$

Symmetric correlator: $K_{\perp s}(\vec{q},\omega) = \dfrac{\pi T}{\hbar^2 \varepsilon_q}\left\{\delta(\omega - \varepsilon_q) + \delta(\omega + \varepsilon_q)\right\},$

Antisymmetric correlator: $K_{\perp a}(\vec{q},\omega) = \dfrac{\pi T}{\hbar^2 \varepsilon_q}\left\{\delta(\omega - \varepsilon_q) - \delta(\omega + \varepsilon_q)\right\},$

$\hbar\varepsilon_q = J_q - J_0 = D\hbar^2 q^2 = \dfrac{\hbar^2 q^2}{2 m_{SW}}$ is SW spectrum, SW stiffness $D = \dfrac{1}{2} J_0 \bar{r}^2,$

$\bar{r}^2 = \int d\vec{r}\, r^2 J(r) / J_0,\ J_0 = \int d\vec{r} J(r)$ is exchange integral

$D\hbar^2 q^2 \sim \hbar\Omega \cdot (q\bar{r})^2 << \hbar\Omega,$ if $q\bar{r} << 1$

Magnetic scattering cross section

Double differential cross section :

$$\frac{\partial^2 \sigma}{\partial \omega \partial \Omega} = A_m^2 \frac{k_f}{k_i} \left| \frac{1}{2} g F_m(\vec{q}) \right|^2 \mathbf{S}(\vec{q}, \omega), \quad \text{where } A_m^2 = \left(\frac{\gamma e^2}{m_e c^2} \right)^2 = 0.292 \text{ barn},$$

$\vec{q} = \vec{k}_f - \vec{k}_i$ is the wave vector and $\hbar\omega = E_f - E_i$ energy transfers

Intermediate scattering function:

$$\mathbf{S}(\vec{q}, \omega) = \left\{ (\delta^{\alpha\beta} - e^\alpha e^\beta) + i(\varepsilon^{\alpha\beta\gamma} P^\gamma + [\vec{e} \times \vec{P}]^\alpha e^\beta - e^\alpha [\vec{e} \times \vec{P}]^\beta) \right\} K^{\alpha\beta}$$

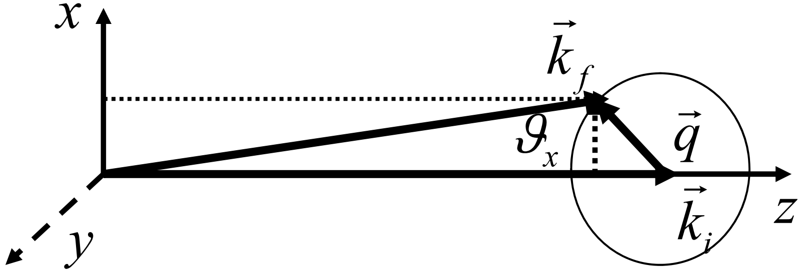
$$\mathbf{S}(\vec{q}, \omega) = [1 + (\vec{e} \cdot \vec{m})^2] K_{\perp s} + 2(\vec{e} \cdot \vec{m})(\vec{e} \cdot \vec{P}) K_{\perp a} + [1 - (\vec{e} \cdot \vec{m})^2] K_{\parallel},$$

$\vec{e} = \vec{q} / q, \quad \vec{m} = \vec{M} / M$, $\vec{M}$ is magnetization,

$\vec{P}$ is polarization vector.

Unpolarized neutron scattering

$$\mathbf{S}(\vec{q},\omega) = \frac{\pi T}{\hbar^2 \varepsilon_q}[1 + (\vec{e}\cdot\vec{m})^2]\left\{\delta(\omega - \varepsilon_q) + \delta(\hbar\omega + \varepsilon_q\right\}$$



$$\hbar\varepsilon_q = D\hbar^2 q^2 + \mu H,\; q^2 = q_x^2 + q_y^2 + q_z^2,$$

$$q_z \approx k_i(\omega/2\varepsilon_i),\quad q_x \approx k_i\vartheta_x,\quad q_y \approx k_i\vartheta_y$$

$$q = |\vec{q}| \approx k_i\sqrt{\vartheta^2 + (\omega/2\varepsilon_i)^2},\quad \vartheta^2 = \vartheta_x^2 + \vartheta_y^2$$

$$\hbar\omega = D\hbar^2 k_i^2[\vartheta^2 + (\omega/2\varepsilon_i)^2] + \mu H$$

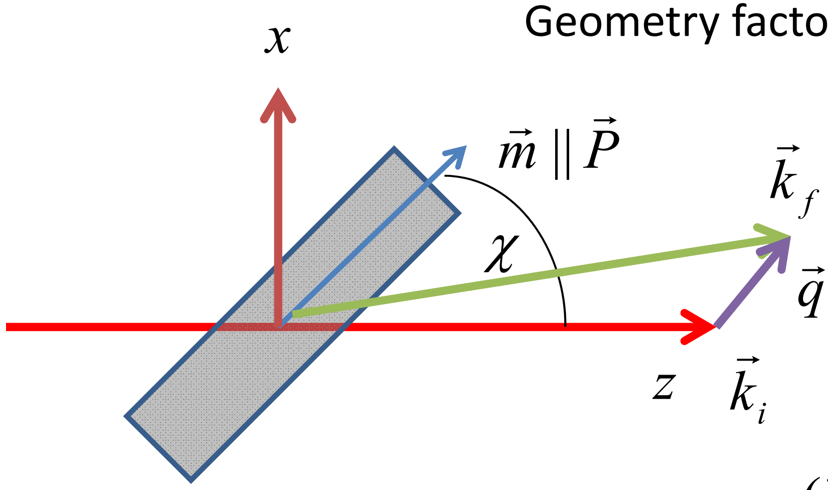
$$\xi = \frac{\hbar\omega}{2E_i},\quad \vartheta_0 = \frac{E_i}{D\hbar^2 k_i^2} = \frac{m_{SW}}{m},\quad h = \frac{\mu H}{D\hbar^2 k_i^2}$$

$$\delta(\omega - \varepsilon_q) = \frac{\vartheta_0}{E}\delta(\xi^2 - 2\xi\vartheta_0 + \vartheta^2 + h)$$

$$\xi^2 - 2\xi\vartheta_0 + \vartheta^2 + h = 0,\quad \xi_\pm = \vartheta_0 \pm \sqrt{\vartheta_0^2 - (\vartheta^2 + h)}$$

scattering is possible only if $-\sqrt{\vartheta_0^2 - h} \le \vartheta \le \sqrt{\vartheta_0^2 - h}$

Geometry factor



Inclined magnetization

$$\hbar\varepsilon_i = E_i, \quad \hbar\omega = E_f - E_i, \quad \vec{q} = \vec{k}_f - \vec{k}_i$$

$$q^2 \approx k_i^2[\vartheta^2 + (\omega/2\varepsilon_i)^2],$$

$$\vartheta^2 = \vartheta_x^2 + \vartheta_y^2, \quad \xi = \omega/2\varepsilon_i$$

$$(\vec{e}\cdot\vec{m})^2 = e_x^2 m_x^2 + e_z^2 m_z^2 + 2e_x e_z m_x m_z$$

$$\approx \frac{\vartheta_x^2 \sin^2\chi + \xi^2\cos^2\chi}{\vartheta^2 + \xi^2} + \frac{2\vartheta_x\xi}{\vartheta^2+\xi^2}\sin 2\chi$$

even function　　odd function

$$\mathbf{S}(\vec{q},\omega) = \frac{\pi T}{(2E_i)^2}\left[1 + \frac{\vartheta_x^2\sin^2\chi + \xi^2\cos^2\chi}{\vartheta^2+\xi^2}\right]\frac{\vartheta_0}{\sqrt{\vartheta_0^2 - (\vartheta^2 + h)}}\frac{1}{|\xi|}$$
$$\times \left\{\delta(\xi - \xi_+) + \delta(\xi - \xi_-) + \delta(\xi + \xi_+) + \delta(\xi + \xi_-)\right\}$$

Antisymmetric scattering function

$$S_a(\vec{q},\omega) = P\,\frac{2\pi T}{\hbar^2 \varepsilon_q}(\vec{e}\cdot\vec{m})^2\left\{\delta(\omega-\varepsilon_q)-\delta(\omega+\varepsilon_q)\right\},$$

$$S_a(\vec{q},\omega) = P\,\frac{\pi T}{2E^2}\,\frac{\vartheta_0}{\sqrt{\vartheta_0^2-(\vartheta^2+h)}}\,\frac{\vartheta_x}{\vartheta^2+\xi^2}\,\frac{\xi}{|\,\xi\,|}$$

$$\times\ \left\{\delta(\xi-\xi_+)+\delta(\xi-\xi_-)+\delta(\xi+\xi_+)+\delta(\xi+\xi_-)\right\}$$

Integration over vertical divergence (energy scan):

$$q_{\|} = \text{const}\ \Rightarrow\ \theta = \text{const} \qquad \vartheta_{y\pm} = \pm\sqrt{2\theta_0\,|\,\xi\,|-\theta^2},\quad 2\theta_0\,|\,\xi\,|\geq \theta^2$$

$$S_a(\vec{q},\omega) = P\,\frac{2\pi T}{\hbar^2\varepsilon_i^2}\,\frac{\theta_0^2(\vartheta_z+\xi)^2}{(\theta^2+\vartheta_y^2)^2}\left\{\delta[2\theta_0\xi-(\theta^2+\vartheta_y^2)]-\delta[2\theta_0\xi+(\theta^2+\vartheta_y^2)]\right\}$$

$$\overline{S}_a^{\,y}(\vec{q},\omega) = P\,\frac{\pi T}{E_i^2}\,\frac{(\sqrt{\theta^2-\xi^2}+\xi)^2}{2\xi\,|\,\xi\,|\sqrt{2\theta_0\,|\,\xi\,|-\theta^2}} \quad @\ \theta^2/(2\theta_0)\leq|\,\xi\,|\leq\theta \ \ \&\ \ 0\leq\theta\leq 2\theta_0$$

$$\overline{S}_a^{\,y}(\vec{q},\omega) = 0 \quad \text{otherwise}$$

Dipole-dipole interaction & damping

$$\hbar\varepsilon_q^d = \sqrt{\hbar\varepsilon_q\{\hbar\varepsilon_q + 4\pi\mu M[1-(\vec{e}\vec{m})^2]\}} \quad \text{anizotropic spectrum}$$

$$\hbar\varepsilon_q = D\hbar^2 q^2 + \mu H,$$

$$H = H_{ex} - 4\pi\hat{N}\vec{M}, \quad \hat{N} \quad \text{is demagnetizing tensor,}$$

$$\text{film plane: } \mathrm{N}_{\|} = 0, \quad \mathrm{N}_{\perp} = 1$$

damping $\Gamma = \Gamma_0 (q\bar{r})^2 \varepsilon_q \ln(q\bar{r})$

$$S_{\mathrm{a}} = -2(\boldsymbol{em})^2 \frac{T}{\omega} \operatorname{Im} \tilde{G}_{\mathrm{a}}(\boldsymbol{q}, \omega),$$

$$\tilde{G}_{\mathrm{a}} \simeq \frac{\langle S\rangle(1+\omega_0 G_1)}{1+\omega_0 e_{\|}^2 G_1} \frac{2\omega}{(\omega + \mathrm{i}\tilde{\Gamma})^2 - \tilde{\varepsilon}^2},$$

$$\tilde{\varepsilon}^2 = \varepsilon_0^2(H)(1-2\Lambda) - (\Gamma\Lambda)^2,$$

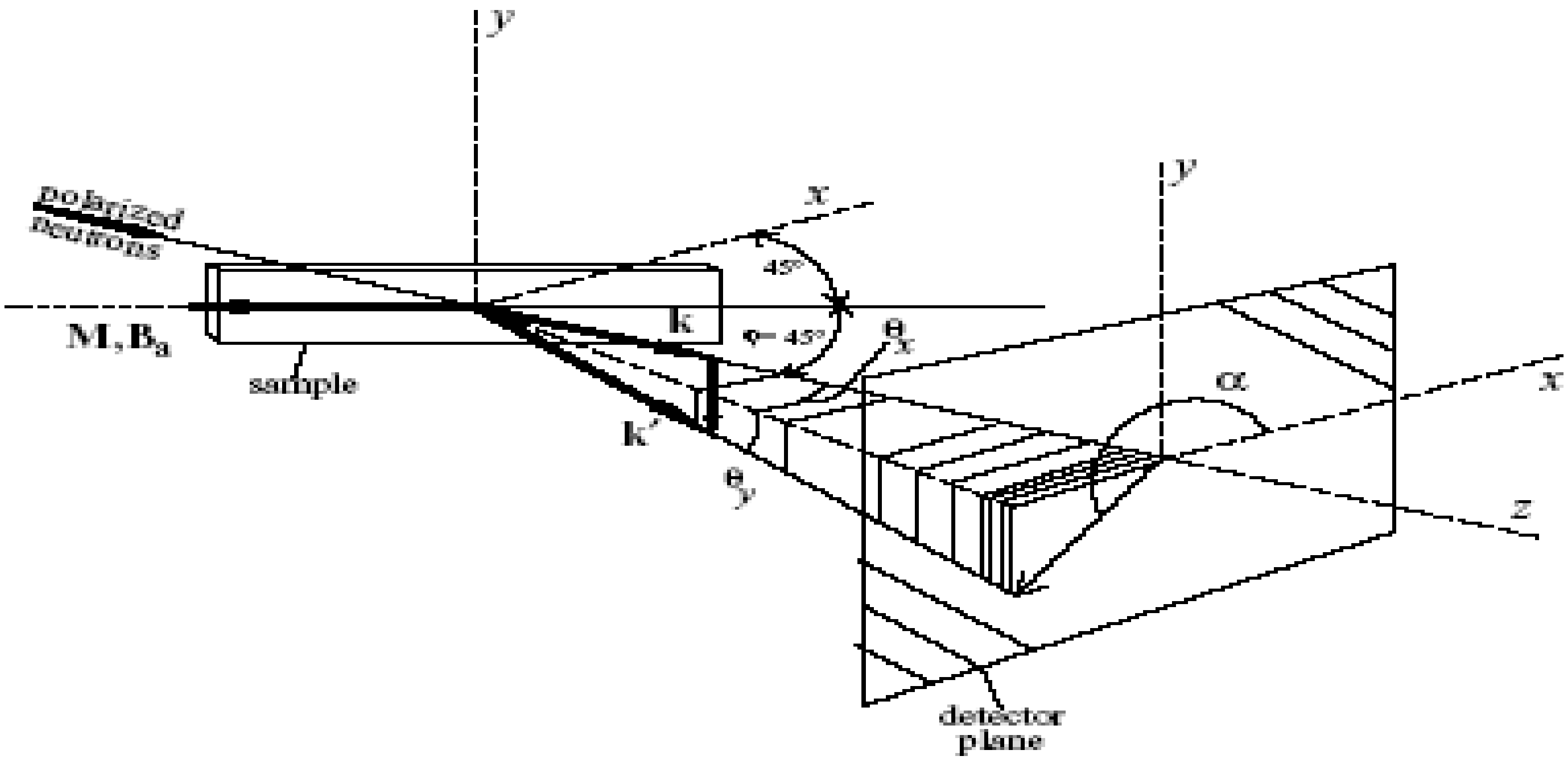
$$\tilde{\Gamma} = \Gamma(1-\Lambda), \qquad \Lambda = \frac{\omega_0 \langle S\rangle (1-e_{\|}^2)}{2\varepsilon_0(H)(1+\omega_0 e_{\|}^2 G_1)}$$

$\omega_0 = 4\pi\mu M$ dipolar energy,

$G_1 = G_{\|}$,

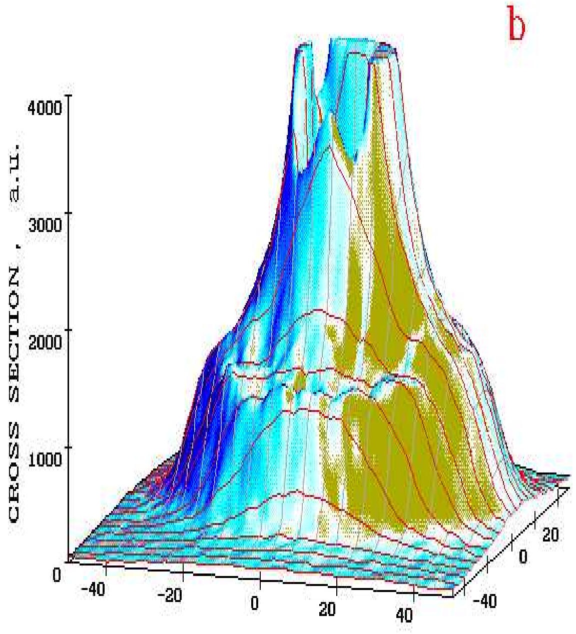
$e_{\|} = (\vec{e}\vec{m})$

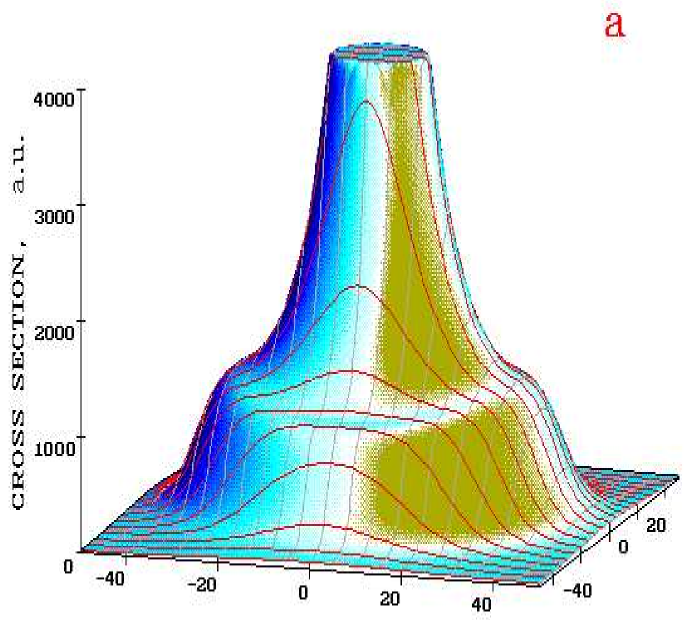
Small Angle Scattering Kinematics



3D Symmetric SW-SANS intensity distribution (short wave length neutrons)

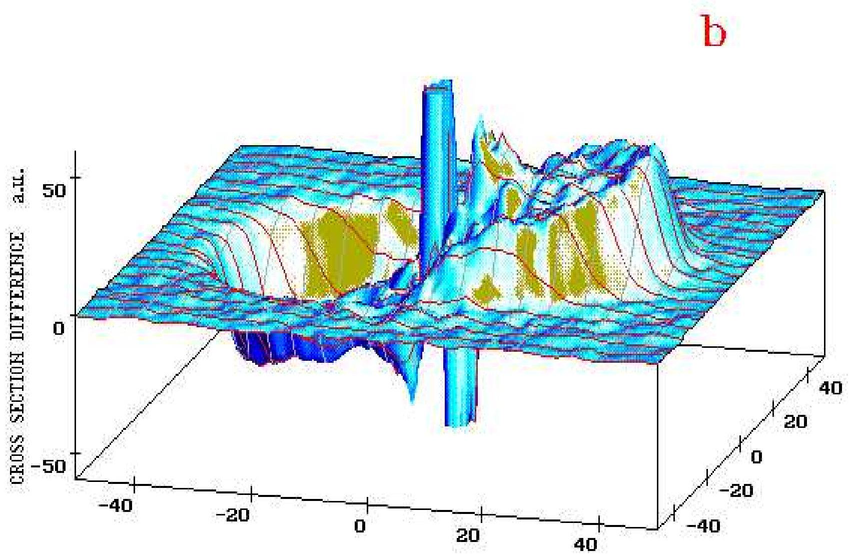
experiment (Okorokov) Fit (Deriglazov)



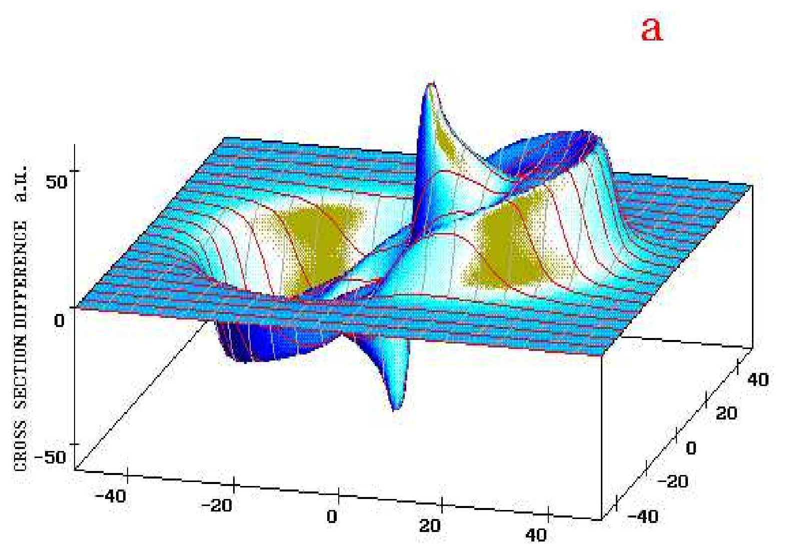


Asymmetry in polarized SW-SANS
(sort wave length neutrons)

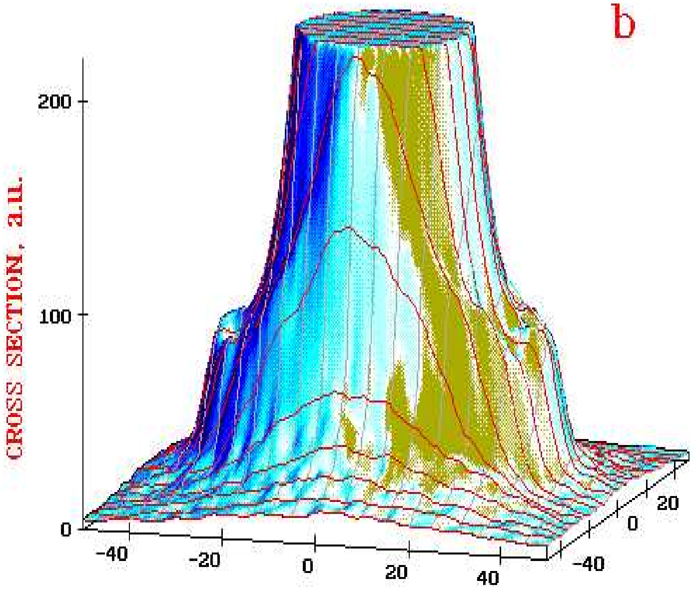
Experiment

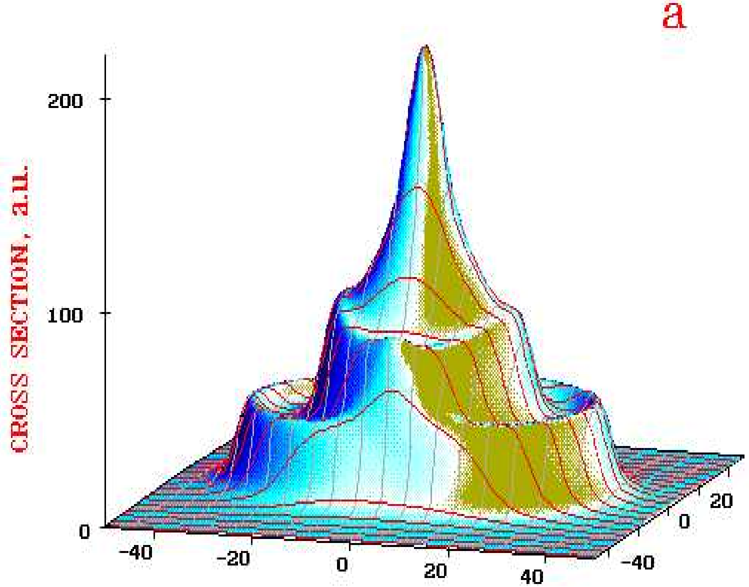


Theory



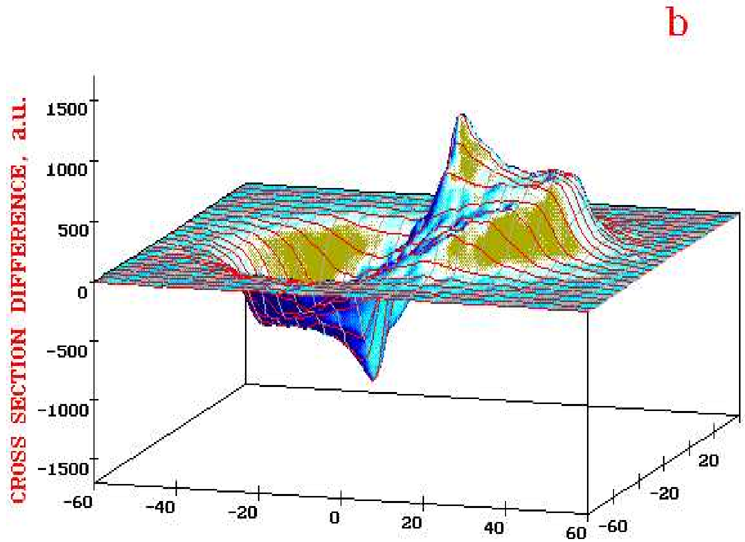
Symmetric part of SW-SANS (long wavelength)

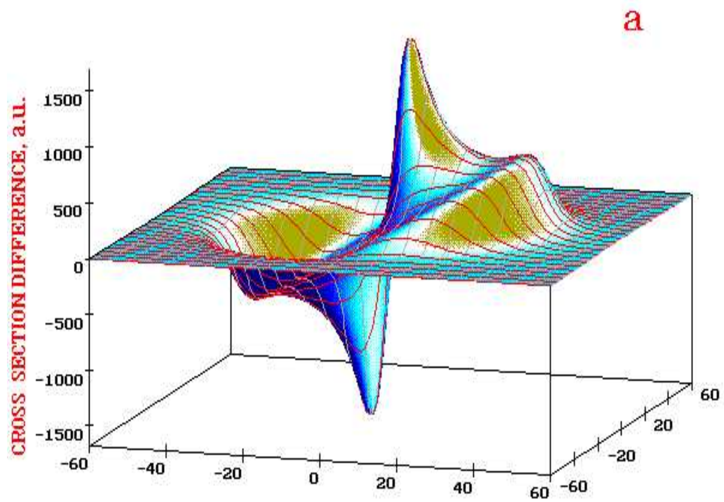




SW-SANS Asymmetry (long wave lenght)

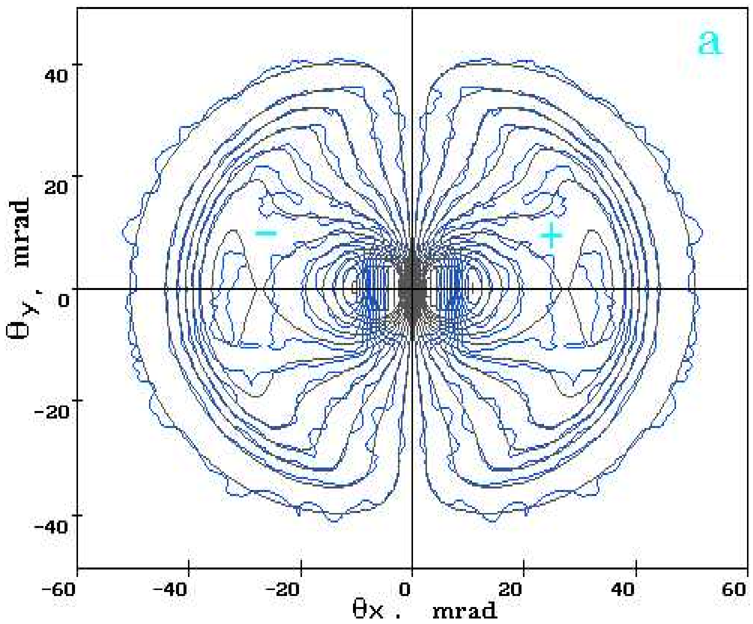
Experiment Theory



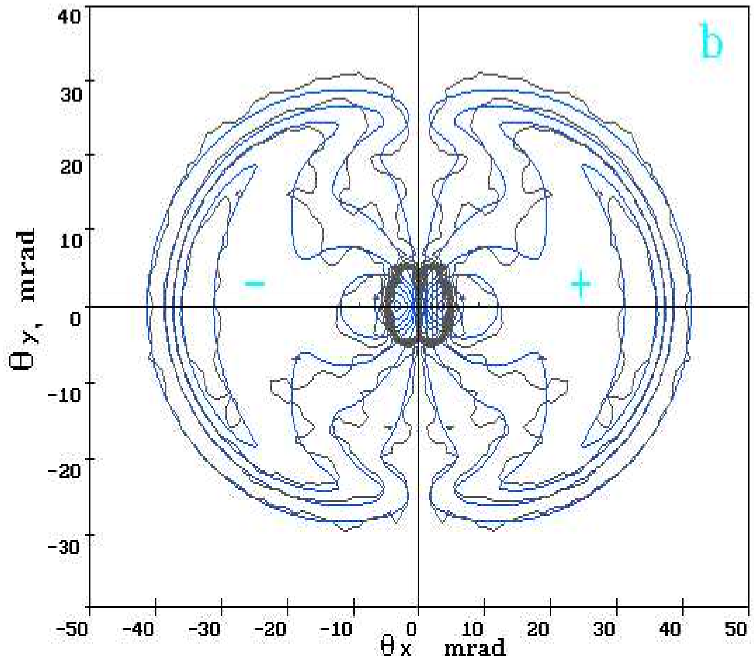


Fit quality illustration

Short wavelength



long wavelength



Sector plots for scattering at neutron wavelength 9.5A

a) horizontal (in the plane of magnetic field)

b) 45 deg.

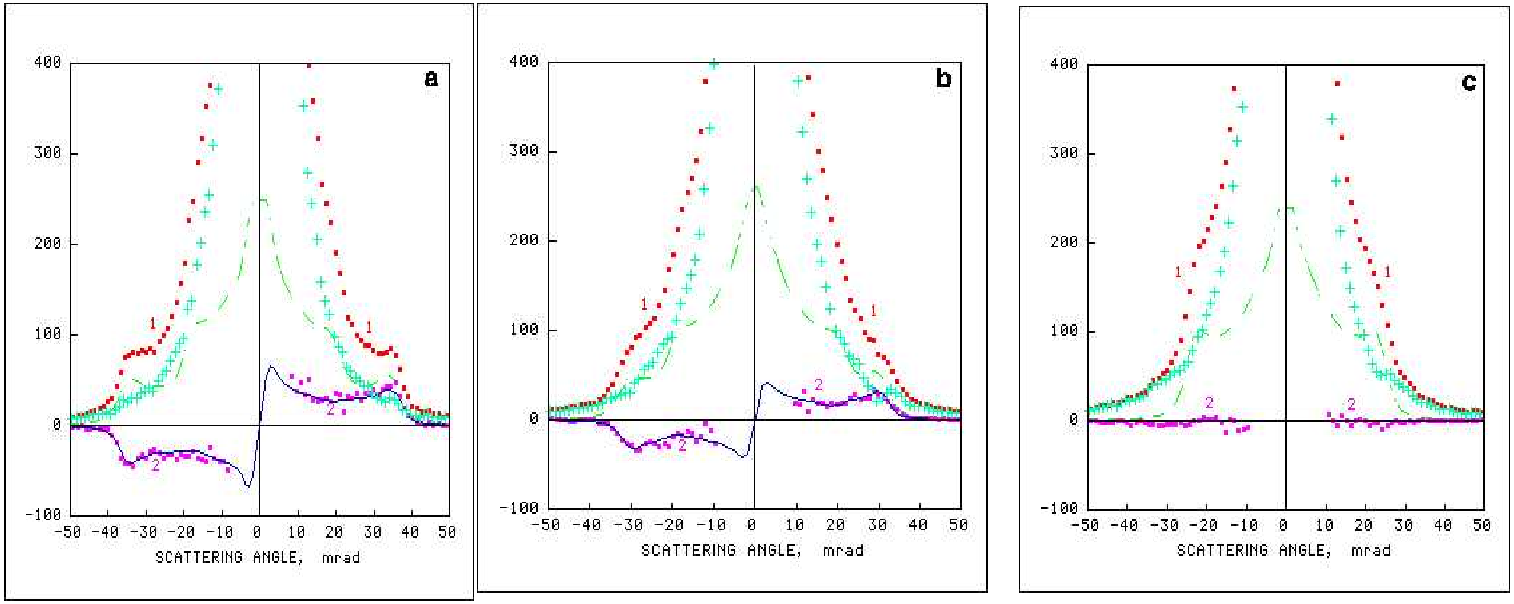
c) and vertical

1 - total scattering,

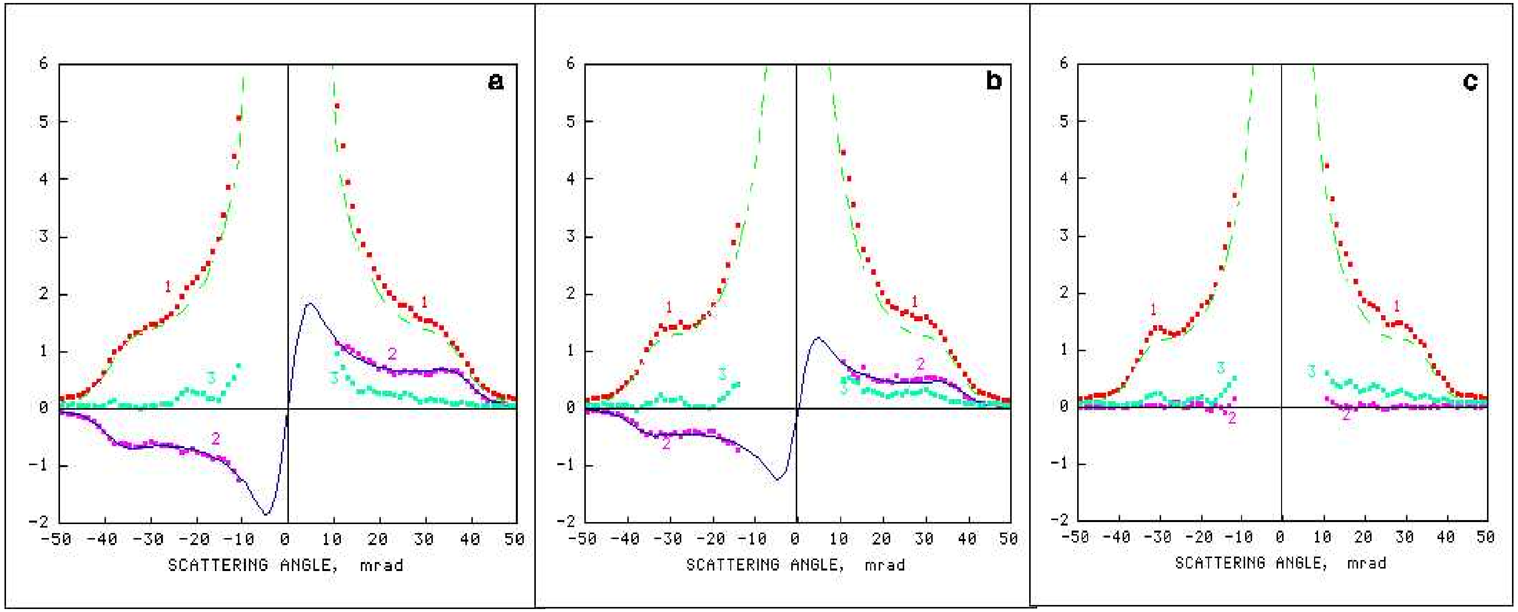
2 – antisymmetrical part: points - experimental data, curves - 2D-best fit,

dashed curve - restored symmetrical part of magnetic scattering,

crosses - residual scattering after subtraction of symmetrical part.



- Sector plots for scattering at neutron wavelength 5.25A for
- A) horizontal ,
- B) 45 deg. vertical
- C) vertical sectors
- 1 - total scattering (points),
- 2 - antisymmetrical part (points and curve are experimental data and 2D-best-fit),
- dashed - restored symmetrical part of magnetic scattering,
- 3 - residual scattering after subtraction of symmetrical part.



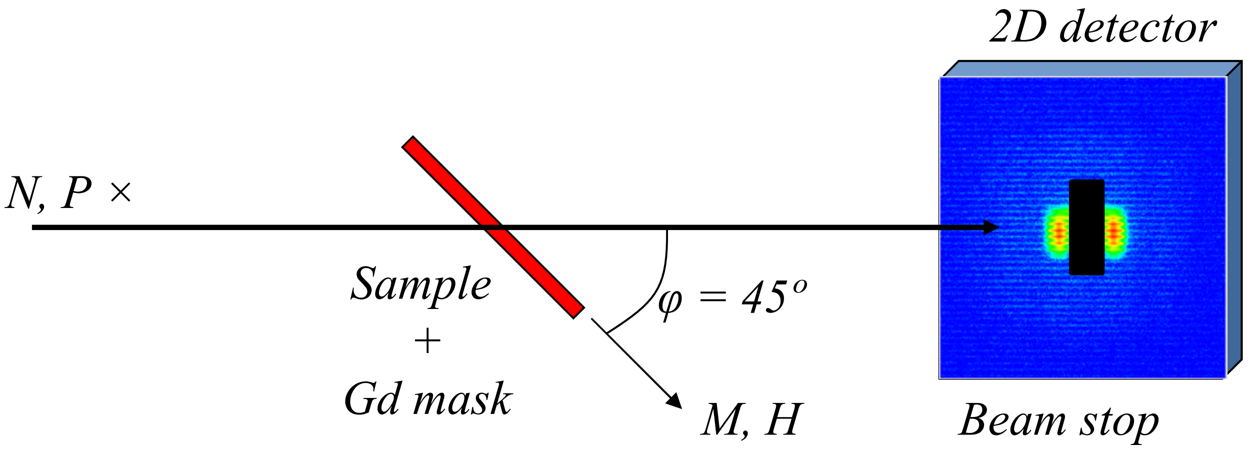
Probing spin waves in amorphous ferromagnetic ribbons by Small Angle Polarized Neutron Scattering

G. A. Badini Confalonieri(1), D. Navas(2), C. Redondo(2), F. Batallan(1), A. Devishvili(3), K. Zhernenkov(3,4), D. Gorkov(3), L.P. Regnault(4), H. Zabel(3) and B.P. Toperverg(3,5)

(1) Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco, 28049 Madrid, Spain
(2) Departamento de Química Física, Univ. del País Vasco, 48940 Leioa, Vizcaya, Spain
(3) Experimentalphysik IV - Festkörperphysik, Ruhr-Universität Bochum, 44780 Bochum, Germany
(4) SPSMS, UMR-E CEA/UJF-Grenoble 1, INAC, Grenoble F-38054, France
(5) Petersburg Nuclear Physics Institute, 188300 Gatchina, Russia

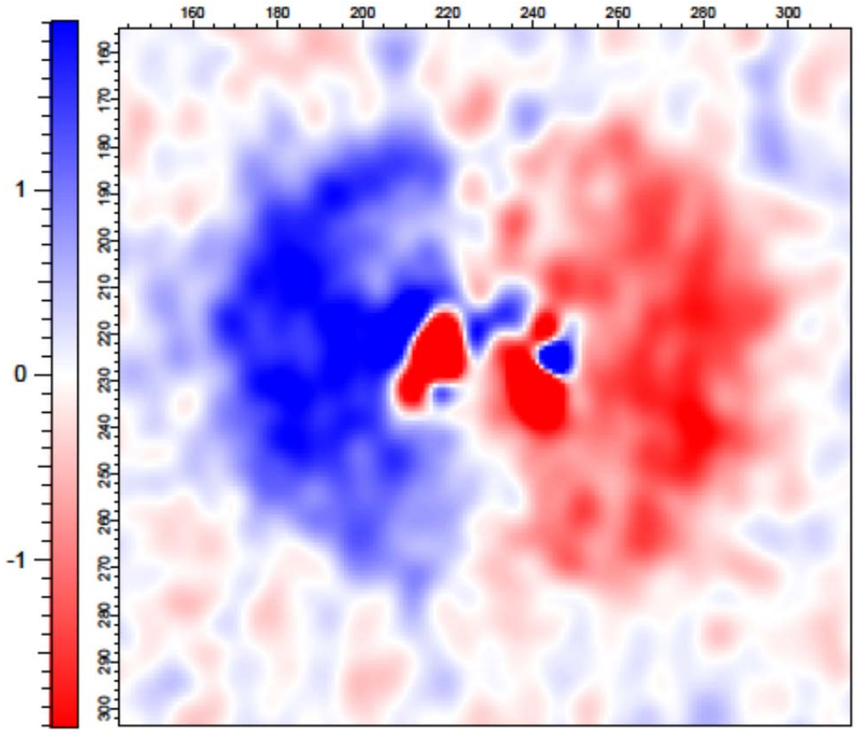
Experimental set-up
SuperADAM, ILL

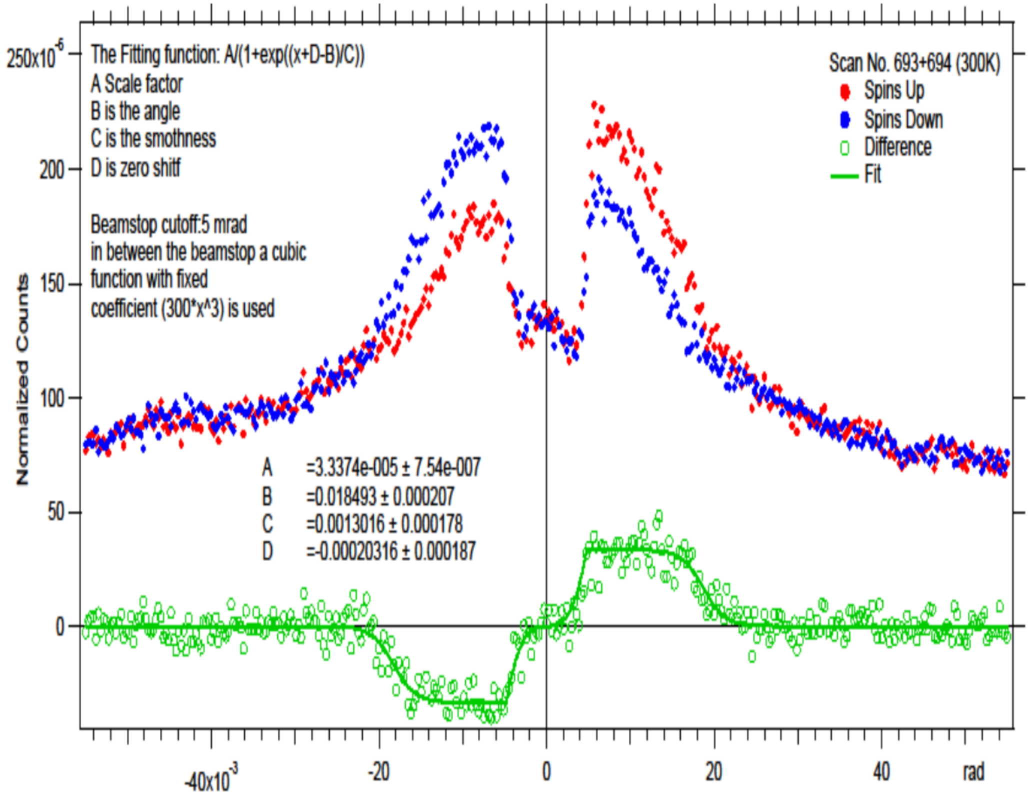
H= 240 Oe
λ = 0.441 nm
Δ T= 300 - 537 K



Sample: $Fe_{40}Ni_{40}Si_{10}B_8Mo_2$

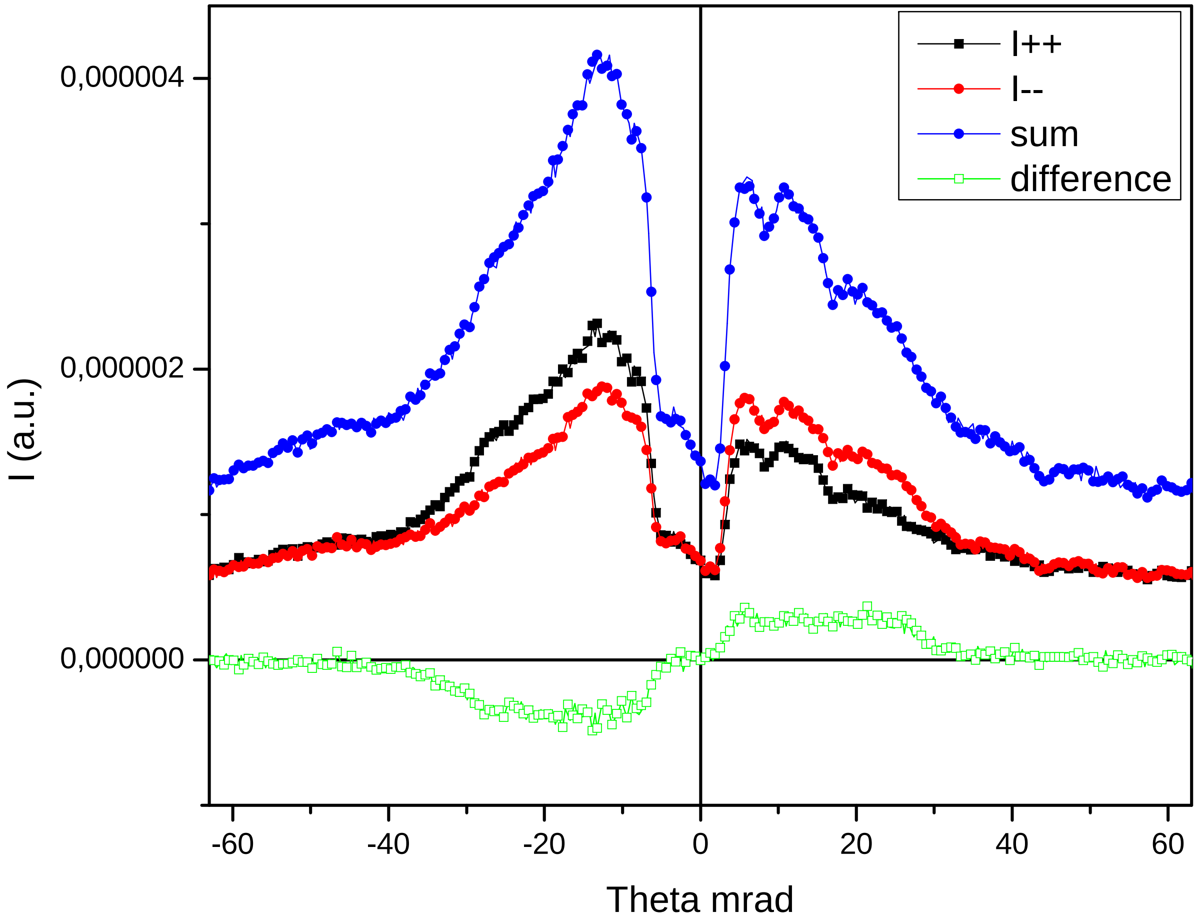
Dimensions: 23 x 24 x 0.026 mm^3 $H_c = 2 \pm 1$ Oe, $T_c = 540$ K





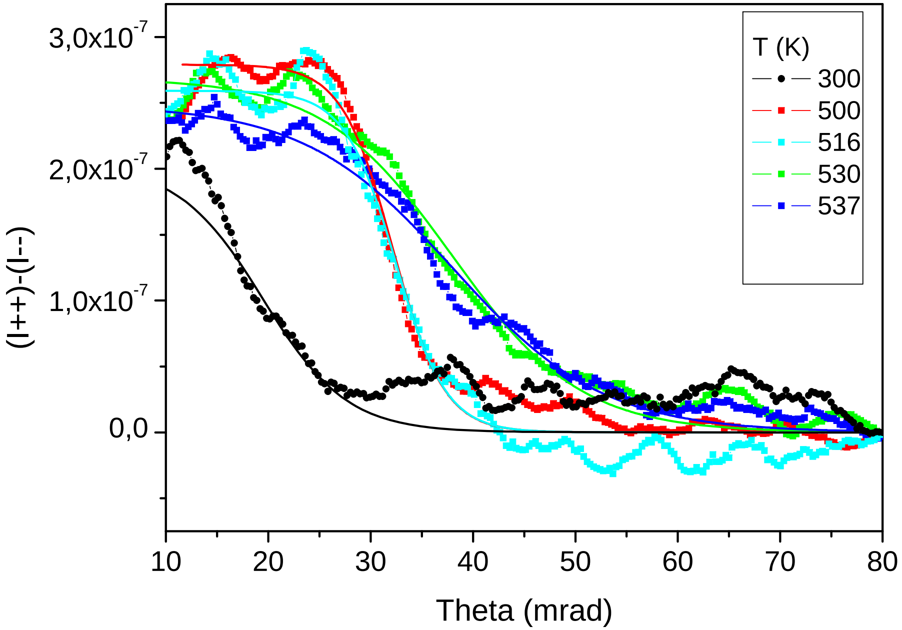
Experimental results

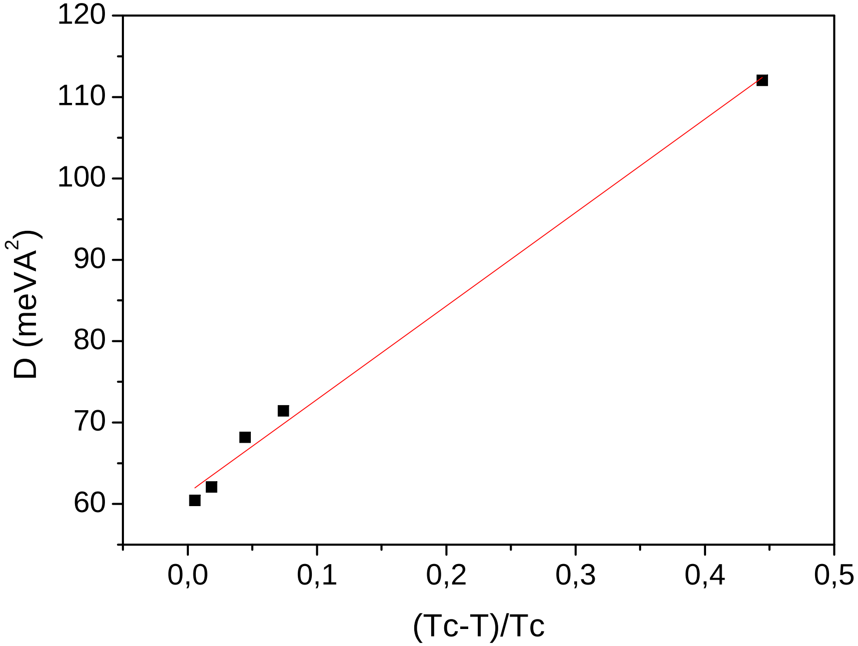
SuperADAM, ILL



Temperature dependence of D

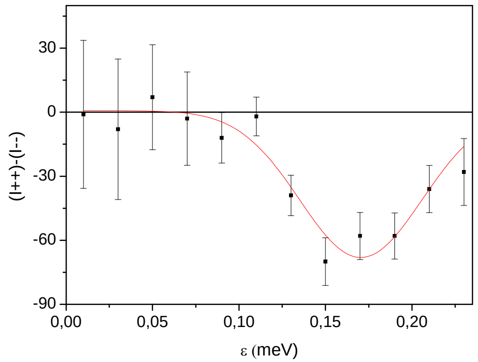
SuperADAM, ILL



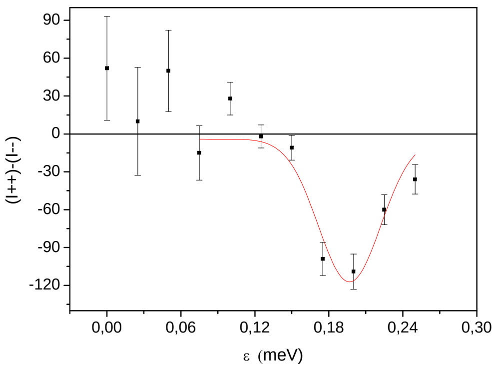


RT inelastic measurement of D

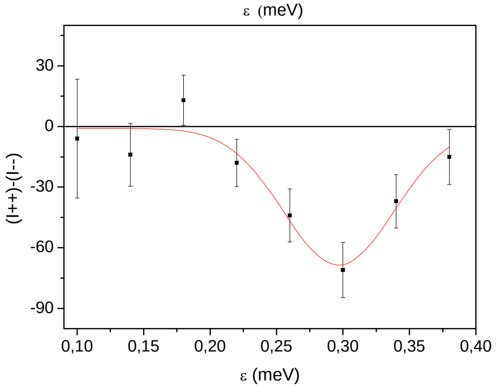
Three axis spectometer IN22, ILL



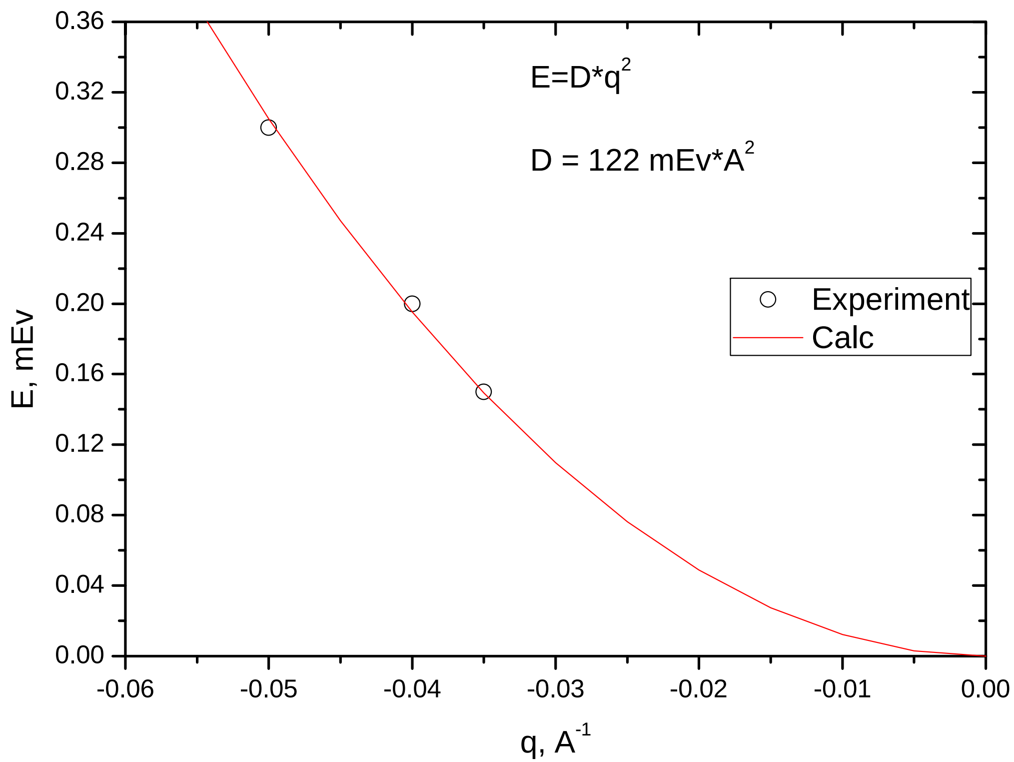
$q = 0.035$ Å^{-1}
$k_i = 1.64$ Å^{-1}



$q = 0.04$ Å^{-1}
$k_i = 1.64$ Å^{-1}



$q = 0.05$ Å^{-1}
$k_i = 1.97$ Å^{-1}

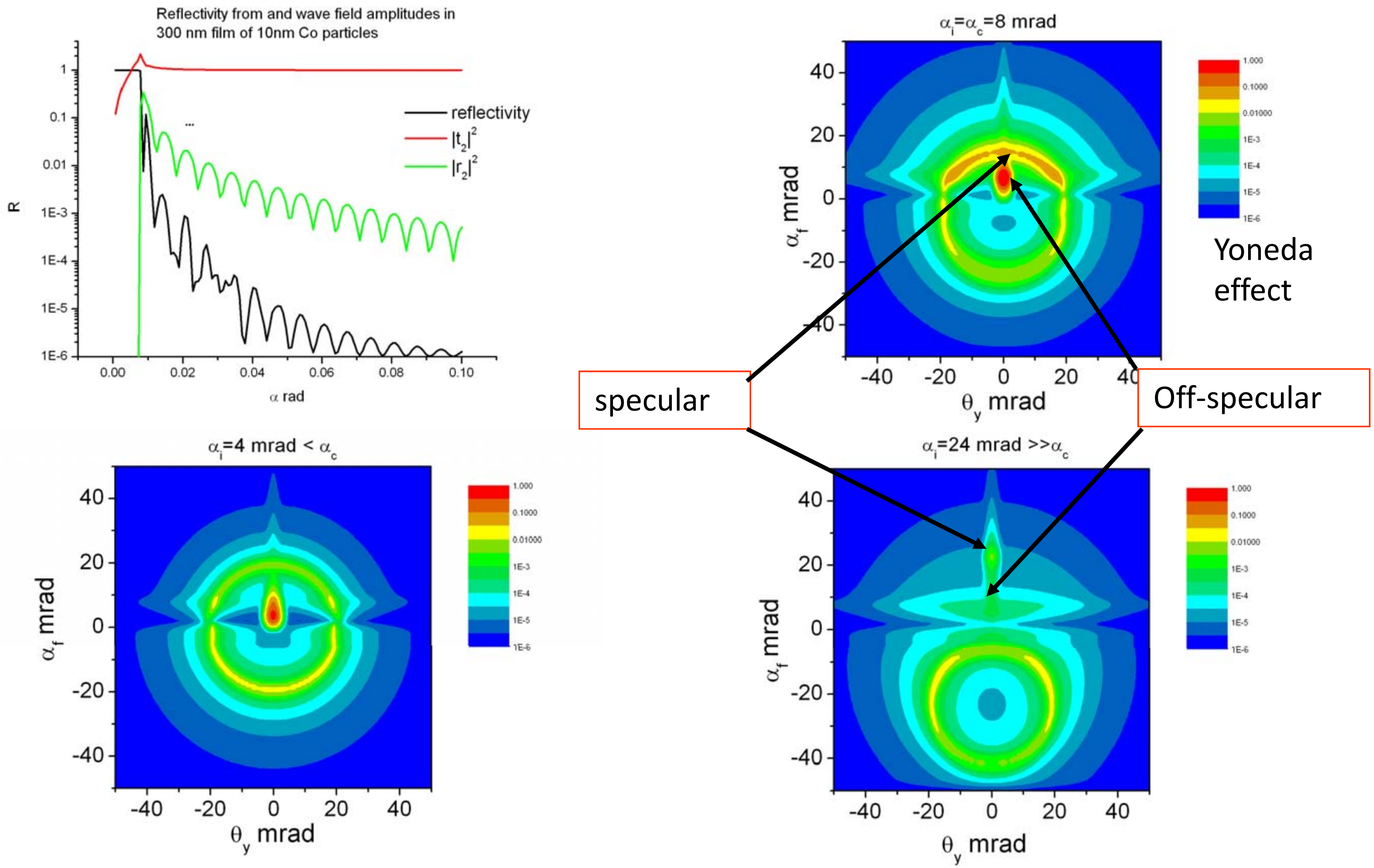


$$\varepsilon = Dq^2$$

Fitted IN22 data result in value of D of 122 meVÅ^2, very close to the RT value obtained at SuperADAM

Grazing incidence small angle diffraction (GISAD)

from 300 nm paracrystalline film composed of 10 nm Co particles



Boris.Toperverg@RUB.de

GISANS Kinematics

$$\kappa_i = k_i \cos\alpha_i = k_i (1 - \alpha_i^2 / 2)$$

$$\kappa_y = k_f \cos\alpha_f \sin\theta_y = k_f (1 - \alpha_f^2 / 2)\theta_y$$

$$\kappa_x = k_f \cos\alpha_f \cos\theta_y = k_f (1 - \alpha_f^2 / 2)(1 - \theta_y^2 / 2)$$

$$\kappa_f = k_i \sqrt{1 + (\varepsilon / E)} \approx k_i (1 + \varepsilon / 2E)$$

$$q_x^2 \approx k_i^2 [(\varepsilon / E) - \theta_y^2 - (\alpha_f^2 - \alpha_i^2)]^2 / 4$$

$$q_y^2 \approx k_i^2 \theta_y^2$$

$$q_{//}^2 = q_x^2 + q_y^2 \approx k_i^2 \{\theta_y^2 + [(\varepsilon / E) - (\alpha_f^2 - \alpha_i^2)]^2 / 4\}$$

Angular singularity in SANS intensity

$$\delta(\varepsilon - Dq_{//}^2) = \frac{1}{\theta_0}\frac{1}{\sqrt{(\theta_0^2-\theta_y^2)+\theta_0(\alpha_f^2-\alpha_i^2)}}\{\delta(\varepsilon-\varepsilon_+)+\delta(\varepsilon-\varepsilon_-)\}$$

$$\varepsilon_\pm = 2E\{\theta_0 + \frac{1}{2}(\alpha_f^2-\alpha_i^2)\pm\sqrt{(\theta_0^2-\theta_y^2)+\theta_0(\alpha_f^2-\alpha_i^2)}\}$$

$$\theta_0 = \frac{E}{Dk^2} >> (\alpha_f^2-\alpha_i^2)$$

$$\varepsilon_\pm = 2E\{\theta_0 \pm\sqrt{\theta_0^2-\theta_y^2}\}$$

GISANS in DWBA

$$\frac{d\sigma}{d\omega} \propto \frac{1}{\theta}\frac{1}{\sqrt{\theta_0^2-\theta_y^2}}|T_i(\alpha_i,E)|^2|T_f(\alpha_f,E+\varepsilon_\pm)|^2|F(p_i,p_\pm)|^2$$

$$T_i = \frac{2\alpha_i}{\alpha_i+\sqrt{\alpha_i^2-\alpha_{ic}^2}}$$

$$T_f = \frac{2\alpha_f}{\alpha_f+\sqrt{\alpha_f^2-\alpha_{fc}^2}}$$

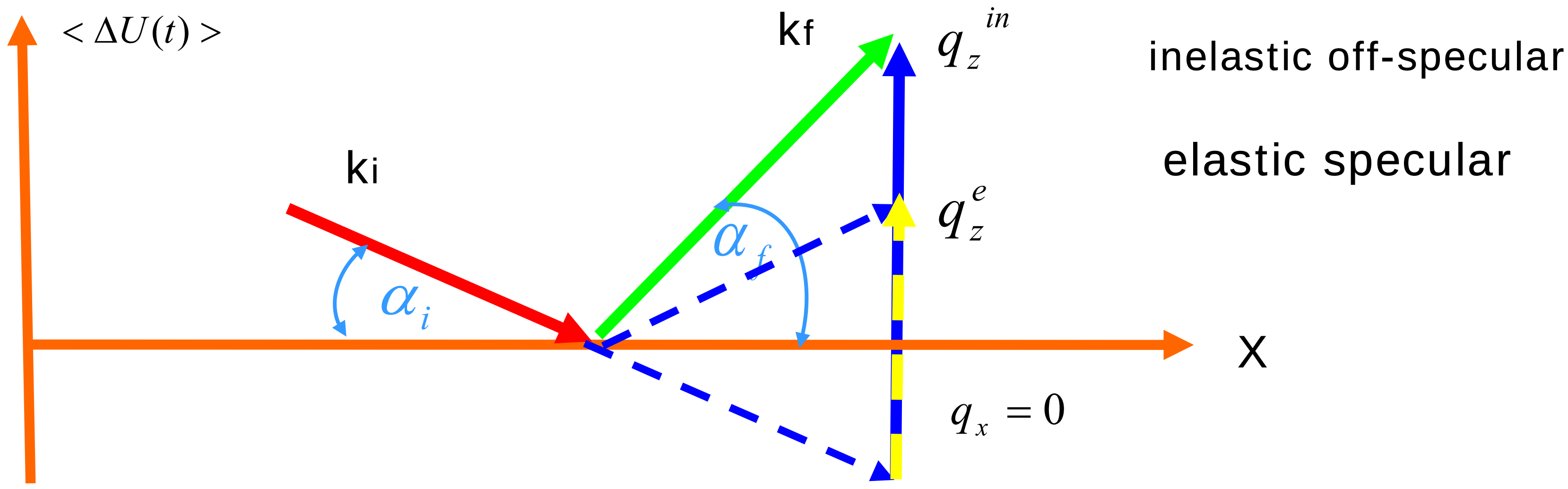
$$\alpha_{ic}^2 = \frac{Nb}{\pi}\lambda_i^2$$

$$\alpha_{fc}^2 = \frac{Nb}{\pi}\lambda_f^2$$

$$\alpha_{fc} \approx \alpha_{ic}(1-\frac{\varepsilon}{2E})$$

Inelastic off-specular reflection on dynamic homogeneous deviations

ΔU(t)=U(t)-U

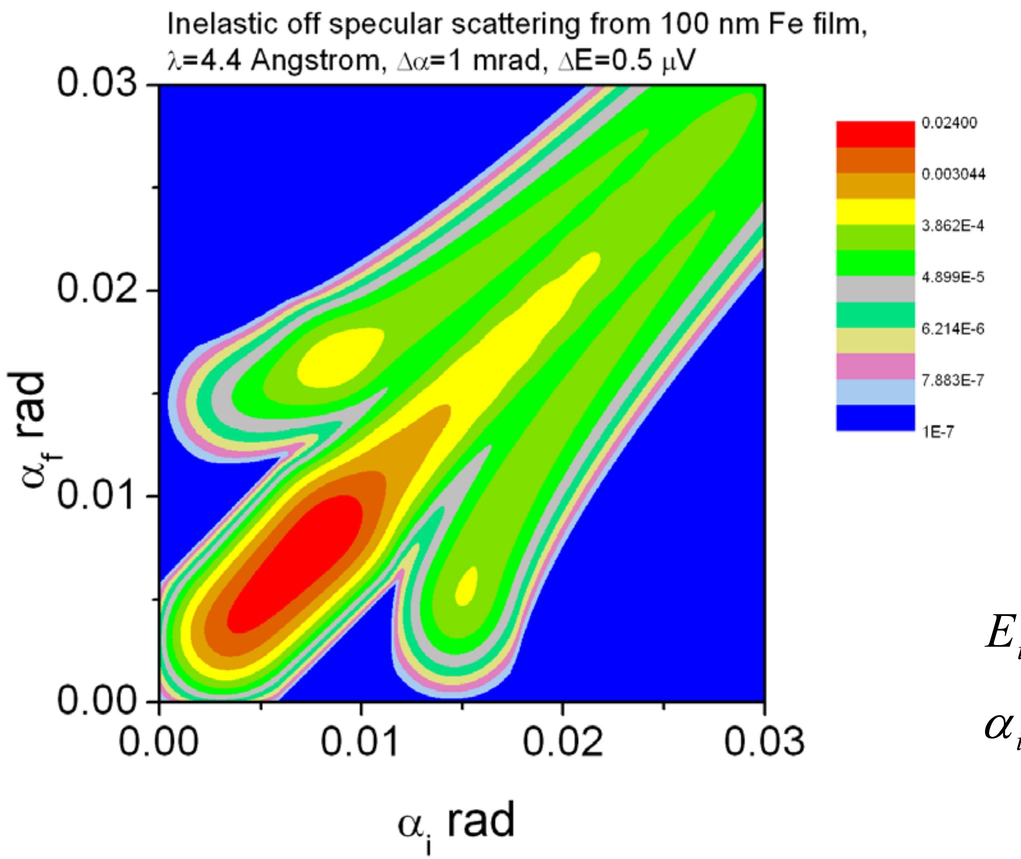


Dynamical fluctuations with size greater coherence length

Inelastic off specular scattering from homogeneous mode

$$E_f = E_i + \Delta E$$

$$\alpha_f = \sqrt{\alpha_i + \Delta E} / E_i$$

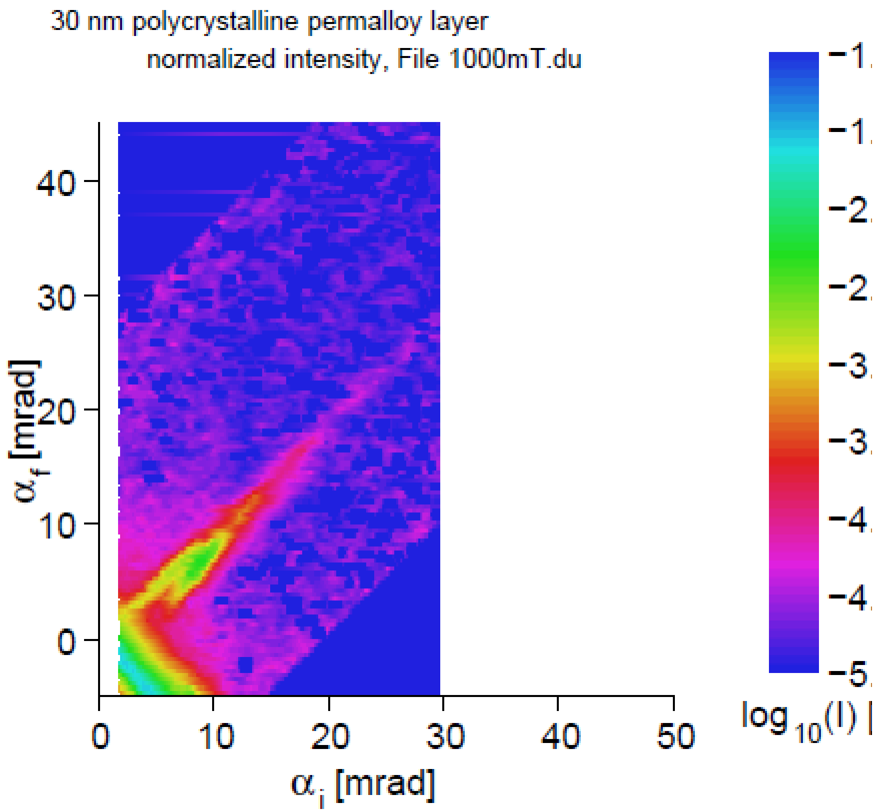


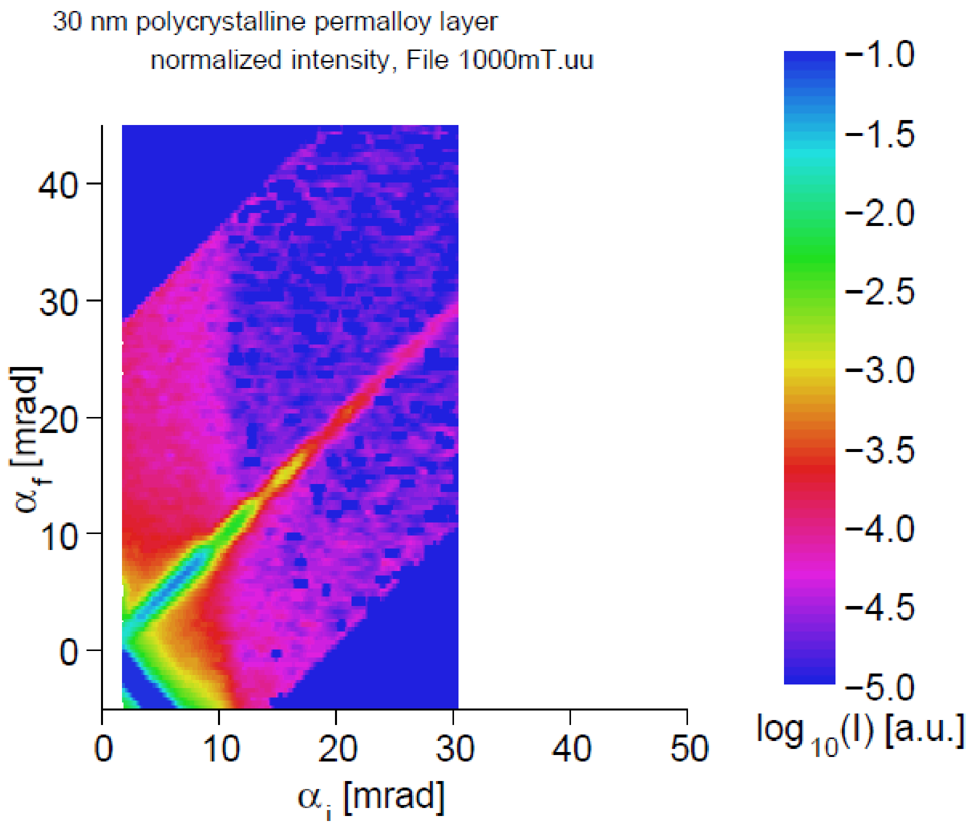
$$E_i = E_f \pm \Delta E$$

$$\alpha_i = \sqrt{\alpha_f + \Delta E} / E_f$$

Boris.Toperverg@RUB.de

U. Ruecker, Juelich





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