Скирмион – две концепции одной частицы (Топология смотрит на нас)

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ПИЯФ НИЦ КИ

в соавторстве с

Ю.В. Барамыгина, В.Е.Тимофеев

06.06.2024 «Спектрина-2024»

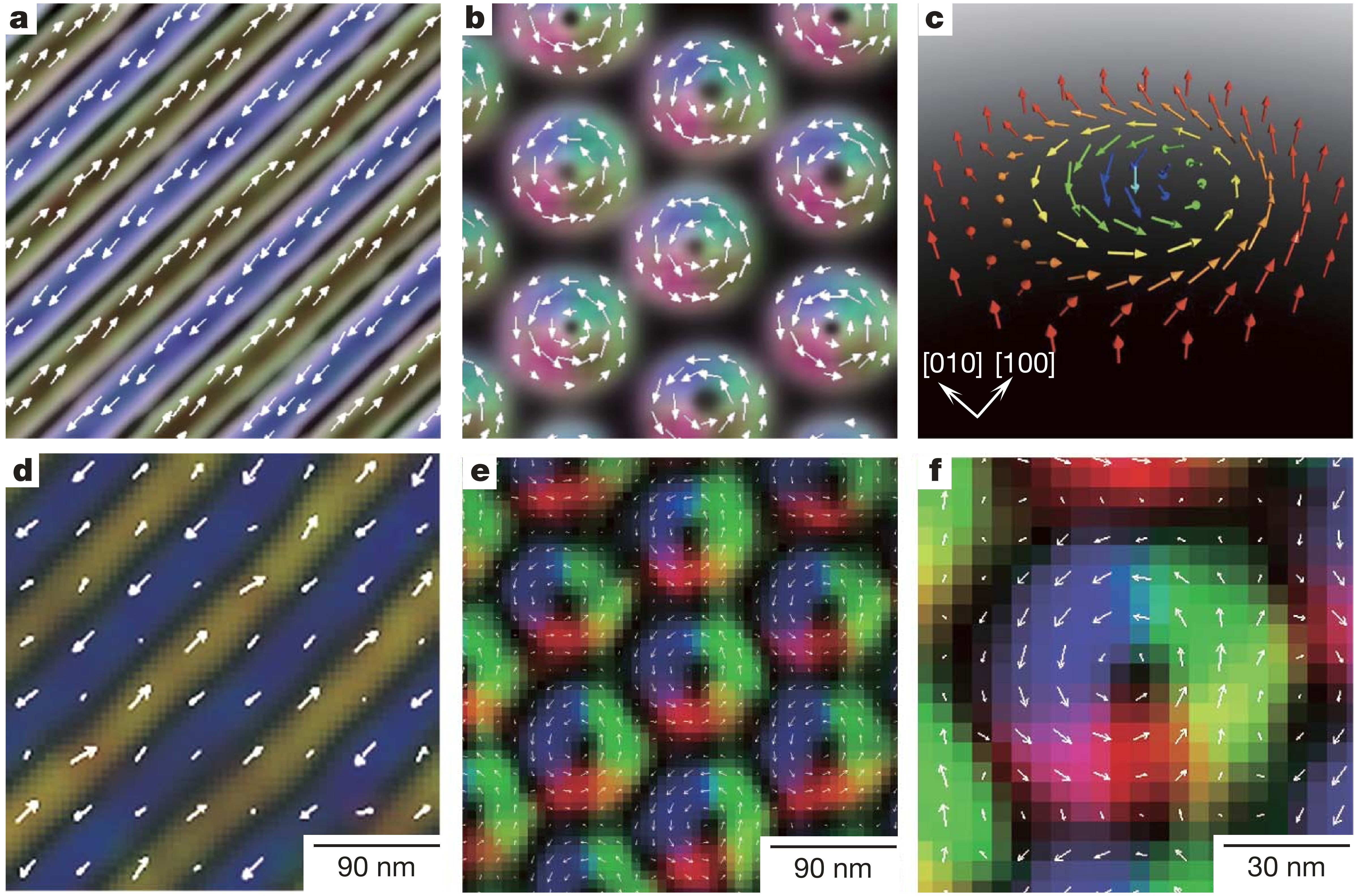
early works

Skyrme, «*A unified field theory of mesons and baryons*», Nuclear Physics, 31, 556(1962)

Belavin, Polyakov, «*Metastable states of two-dimensional isotropic ferromagnets*», JETP Lett. 22, 245 (1975)

Bogdanov, Yablonsky, «*Thermodynamically stable "vortices" in magnetically ordered crystals. The mixed state of magnets*» Sov.Phys. JETP 68, 101 (1989)

skyrmion crystals



theory

experiment
TEM

Yu et al., Nature (2010)

0 T

50 mT

helical magnet Fe0.5Co0.5Si

Melting of Skyrmion crystal

VOLUME 66, NUMBER 21 PHYSICAL REVIEW LETTERS 27 MAY 1991

Hexatic-to-Liquid Melting Transition in Two-Dimensional Magnetic-Bubble Lattices

R. Seshadri and R. M. Westervelt
Department of Physics and Division of Applied Sciences, Harvard University, Cambridge, Massachusetts 02138

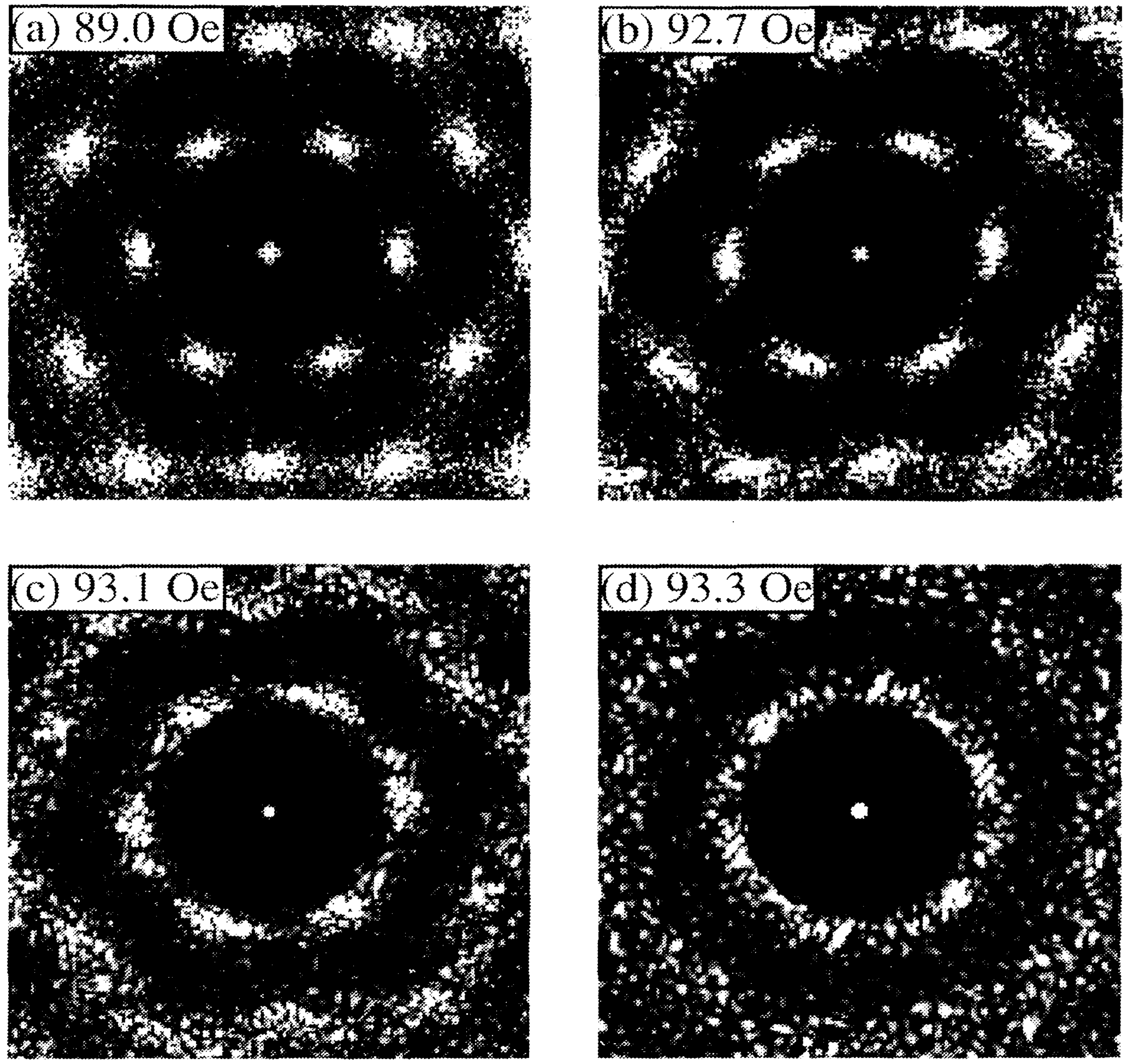


FIG. 2. Two-dimensional structure factor at the magnetic fields H_B indicated (see text).

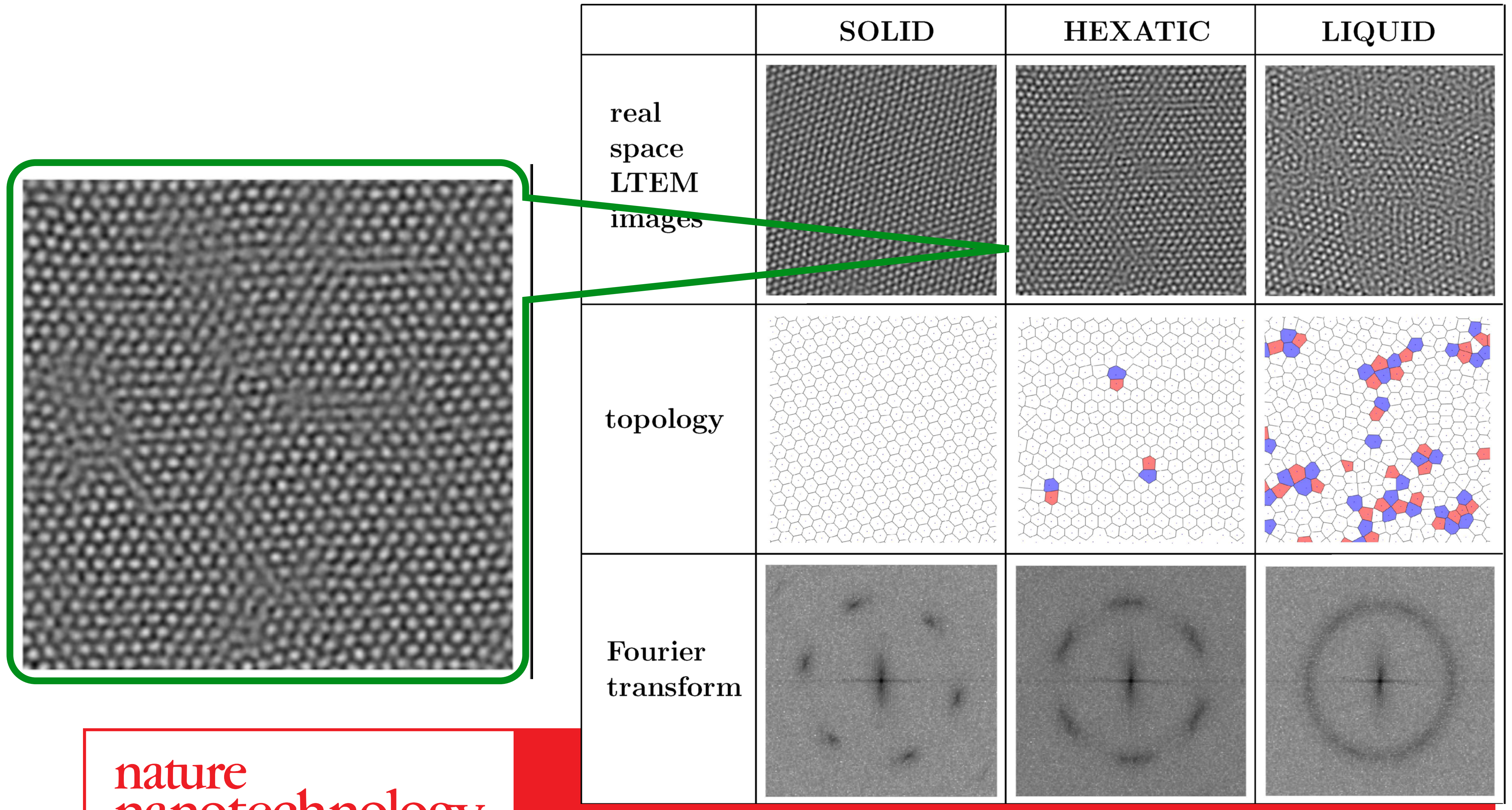
Thin magnetic film of bismuth-substituted iron garnet

Dipolar interaction

Room temperature T= 300 K
Bubble Radius r =3.3 μm
Distance between a=17-47 μm
Film thickness d=7.8 μm

Melting of Skyrmion crystal

NATURE NANOTECHNOLOGY | VOL 15 | SEPTEMBER 2020 | 761–767



nature nanotechnology
https://doi.org/10.1038/s41565-020-0716-3

Melting of a skyrmion lattice to a skyrmion liquid via a hexatic phase

Ping Huang[1,2,3,7], Thomas Schönenberger[2,7], Marco Cantoni[4], Lukas Heinen[5], Arnaud Magrez[6], Achim Rosch[5], Fabrizio Carbone[3] and Henrik M. Rønnow[2]

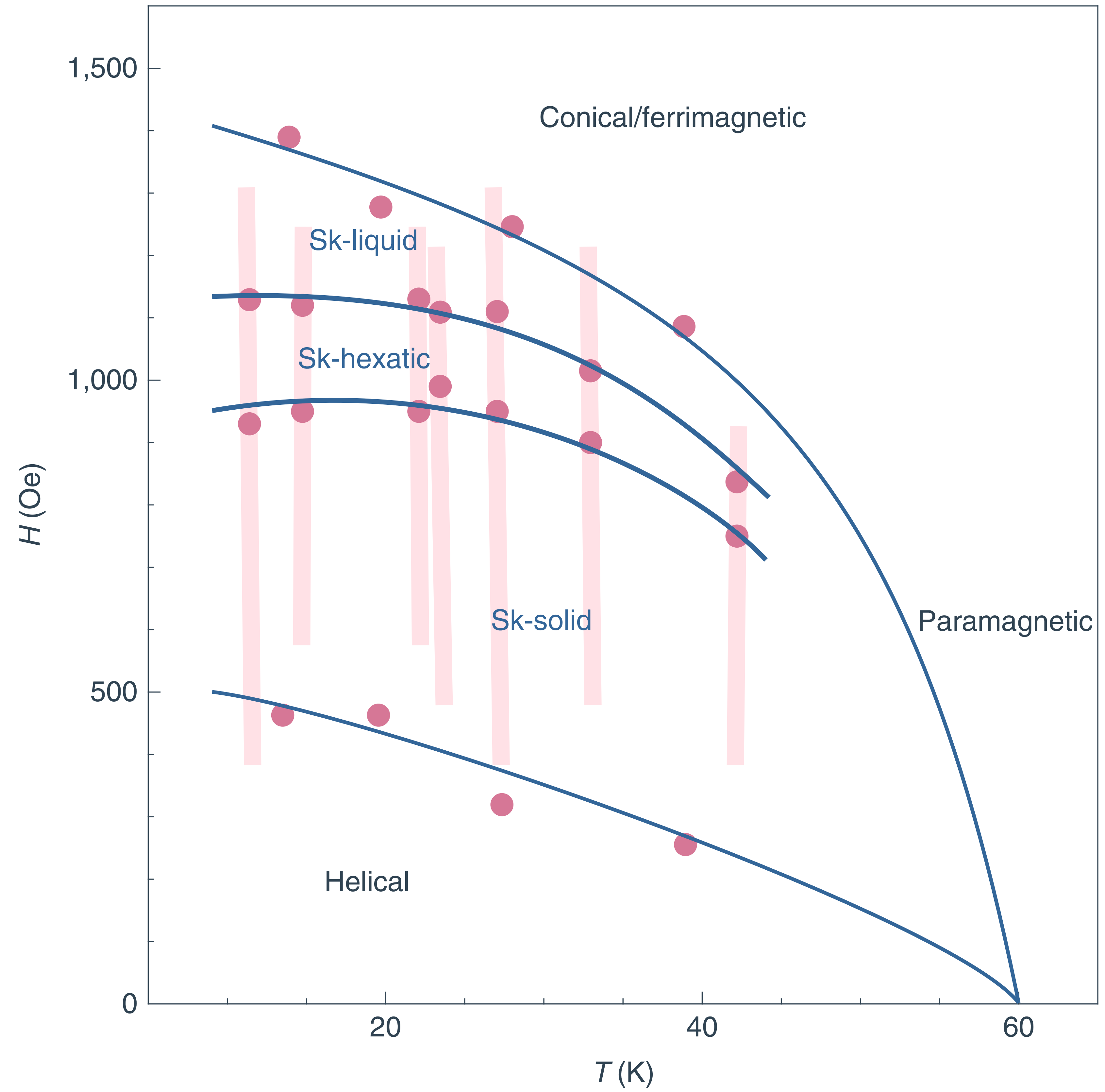


Fig. 5 | Phase diagram of nano-slab Cu_2OSeO_3. Quantitative analysis of LTEM data reveals two new skyrmion phases: the hexatic and liquid phases.

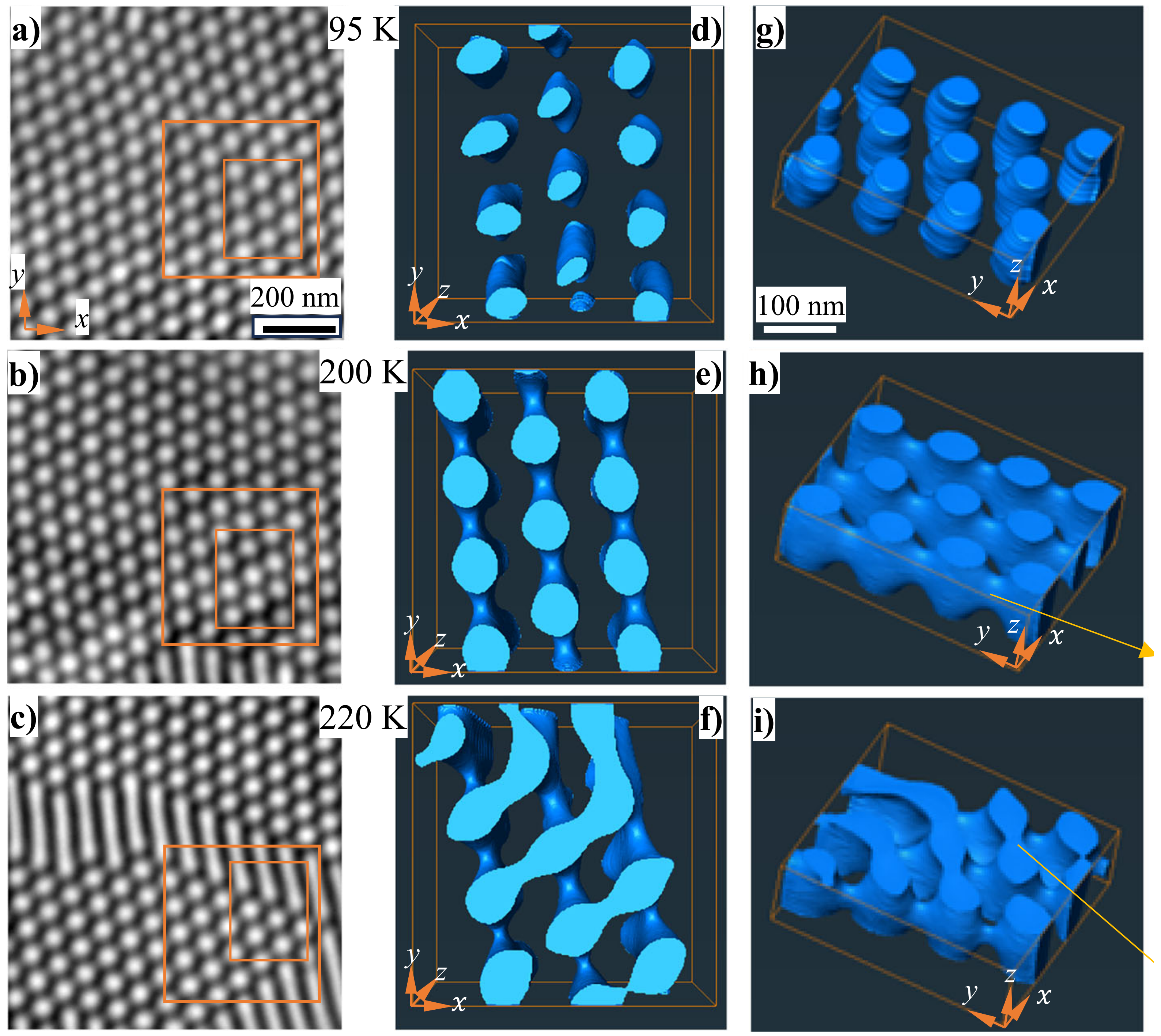
Melting of Skyrmion crystal, 3D

Communications Materials | (2024)5:80

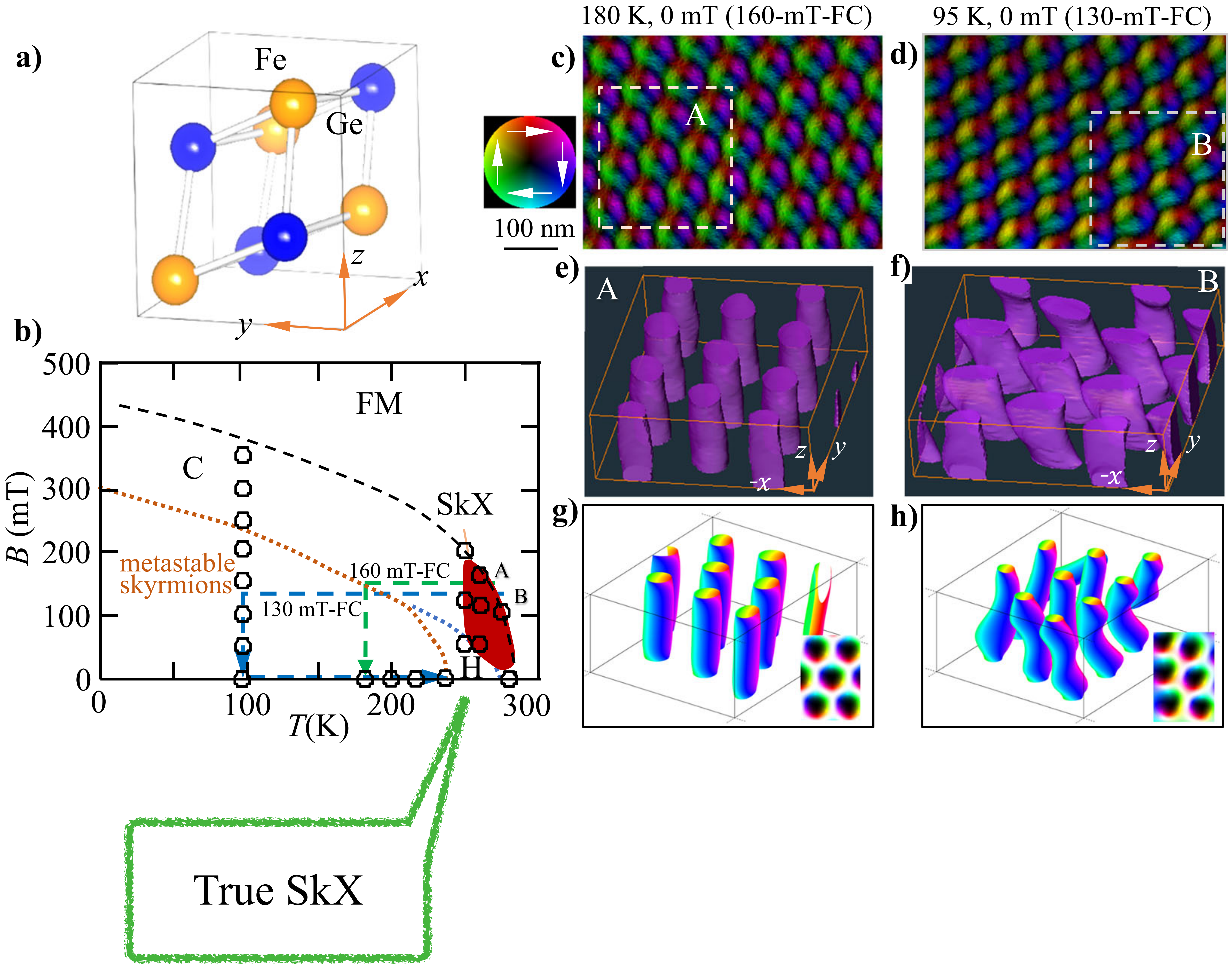
3D skyrmion strings and their melting dynamics revealed via scalar-field electron tomography

Check for updates

Xiuzhen Yu[1], Nobuto Nakanishi[1,2], Yi-Ling Chiew[1], Yizhou Liu[1], Kiyomi Nakajima[1], Naoya Kanazawa[3], Kosuke Karube[1], Yasujiro Taguchi[1], Naoto Nagaosa[1] & Yoshinori Tokura[1,4,5]



FNPP , FeGe

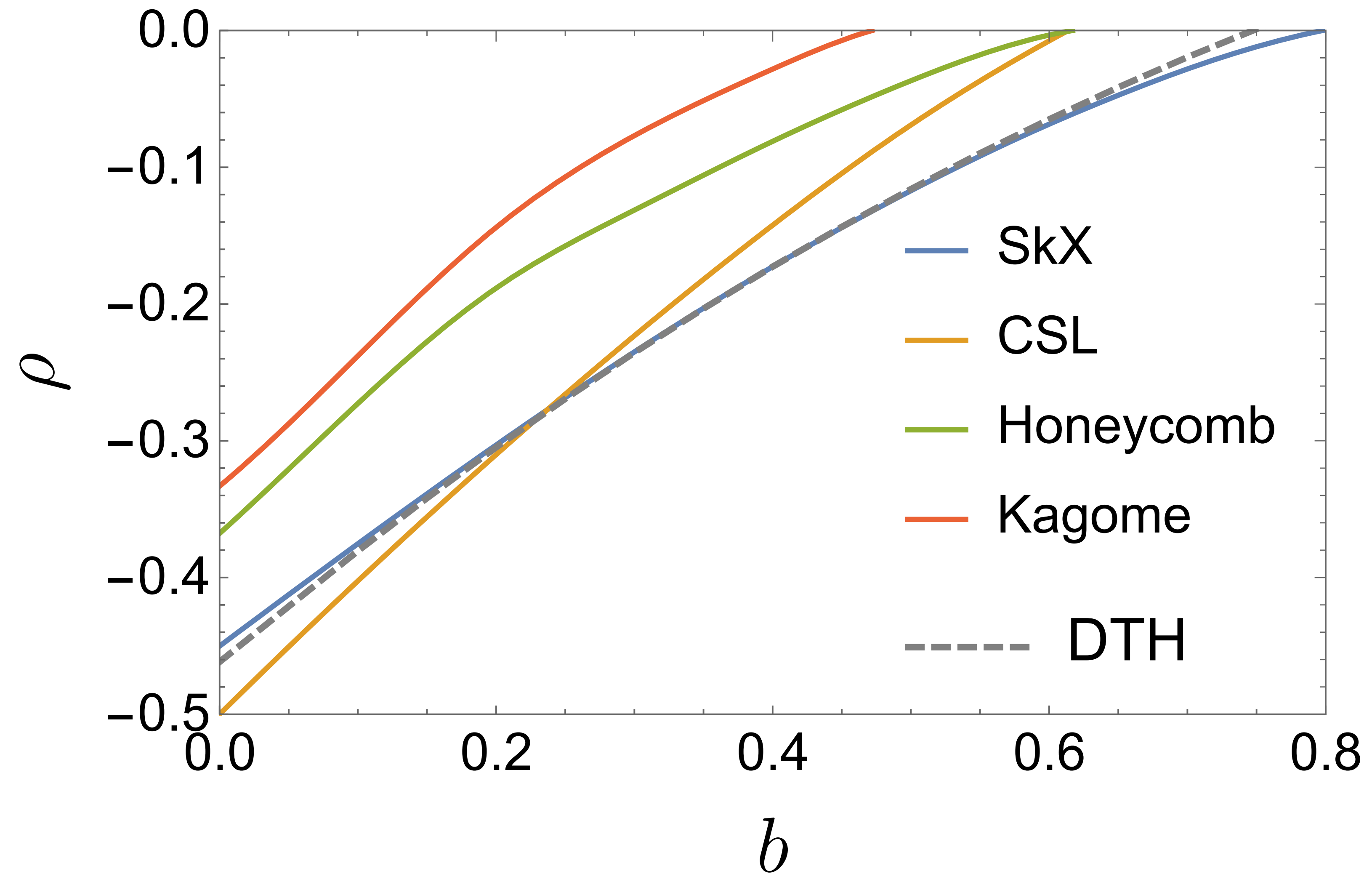


Triple helix versus skyrmion lattice

Timofeev, Sorokin, Aristov
PRB **103**, 094402 (2021)

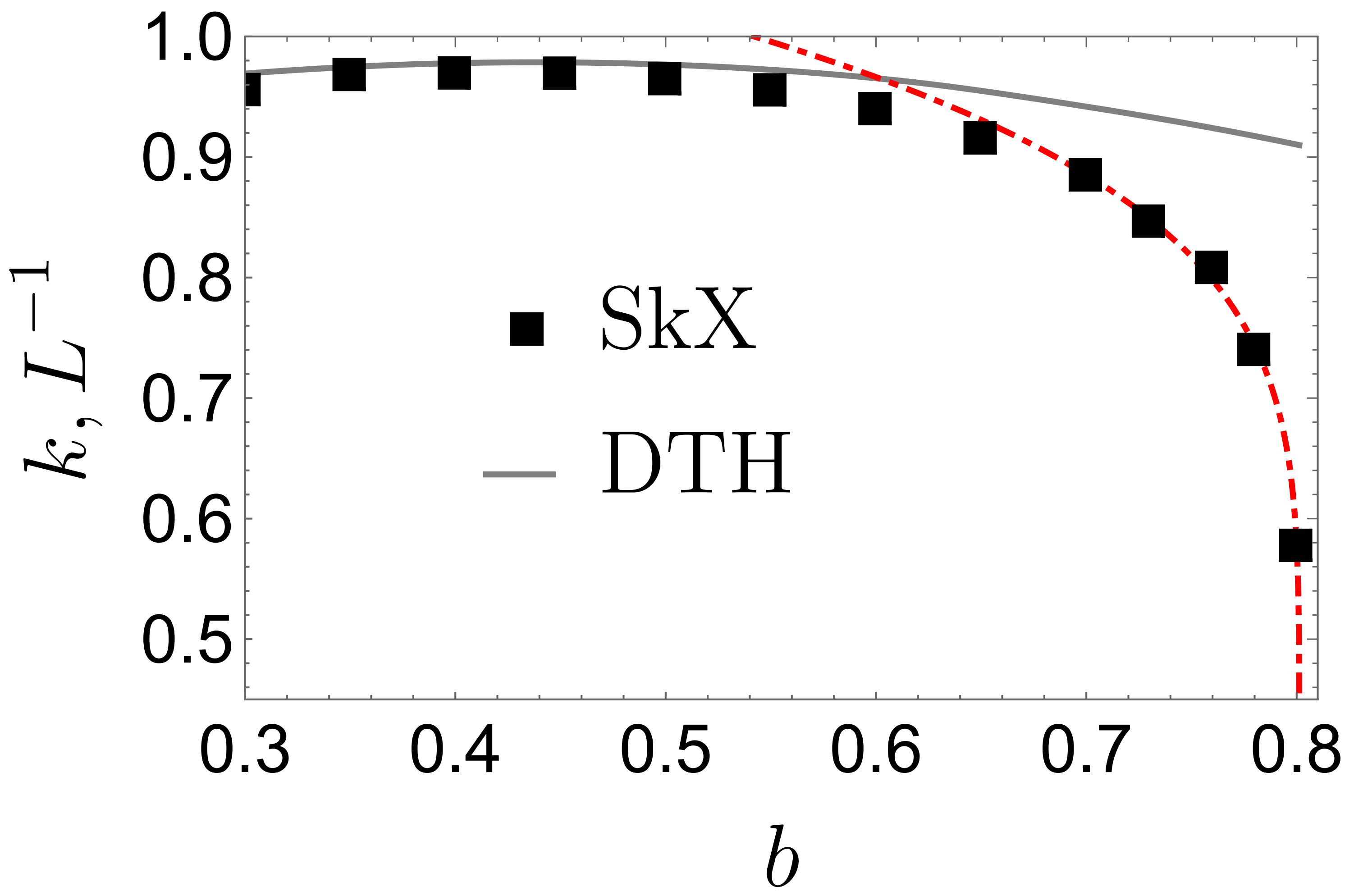
(Deformed) Triple helix

$$\tilde{\mathbf{S}}_{3q} = \frac{S_0\hat{e}_3 + \tilde{\mathbf{S}}_{\varphi=0} + \tilde{\mathbf{S}}_{\varphi=2\pi/3} + \tilde{\mathbf{S}}_{\varphi=4\pi/3}}{|S_0\hat{e}_3 + \tilde{\mathbf{S}}_{\varphi=0} + \tilde{\mathbf{S}}_{\varphi=2\pi/3} + \tilde{\mathbf{S}}_{\varphi=4\pi/3}|}.$$



Skyrmion lattice
(Sum of images of single skyrmions)

$$f(z,\bar{z}) = \sum_j F\left((\bar{z}-\bar{z}_j)/z_0^{(j)}\right)$$



Topological charge:

$$Q = \frac{1}{4\pi}\int d^2\mathbf{r}\ \mathbf{S}\cdot(\partial_x\mathbf{S}\times\partial_y\mathbf{S})$$

Magnon bands in SkX and topology

Schütte, Garst,   Phys.Rev. B (2014)

Roldán-Molina, Núñez, Fernández-Rossier,   New J. Phys. (2016)

M.Garst in «The 2020 skyrmionics roadmap» J. Phys. D: Appl. Phys. (2020)

Dıaz, Hirosawa, Klinovaja, Loss, Phys. Rev. Research (2020)

Two-step procedure

1) Equilibrium local magnetization direction (Monte-Carlo?)
2) Boson representation of spins in local frame

Stereographic projection approach :

Timofeev, Aristov,   Phys.Rev. B (2022)

Minimal continuum model, 2D

$$\mathscr{E} = \frac{1}{2}J(\nabla \mathbf{S})^2 + D\mathbf{S}\cdot\nabla\times\mathbf{S} - B\,S_z$$

Ferromagnetic exchange $J \sim T_c \sim 100$ K

Dzyaloshinskii-Moriya inter. $D \ll J$

Length in units of $l = J/D$ (helix pitch),

Magnetic field in units $D^2/J = H_{c2}$

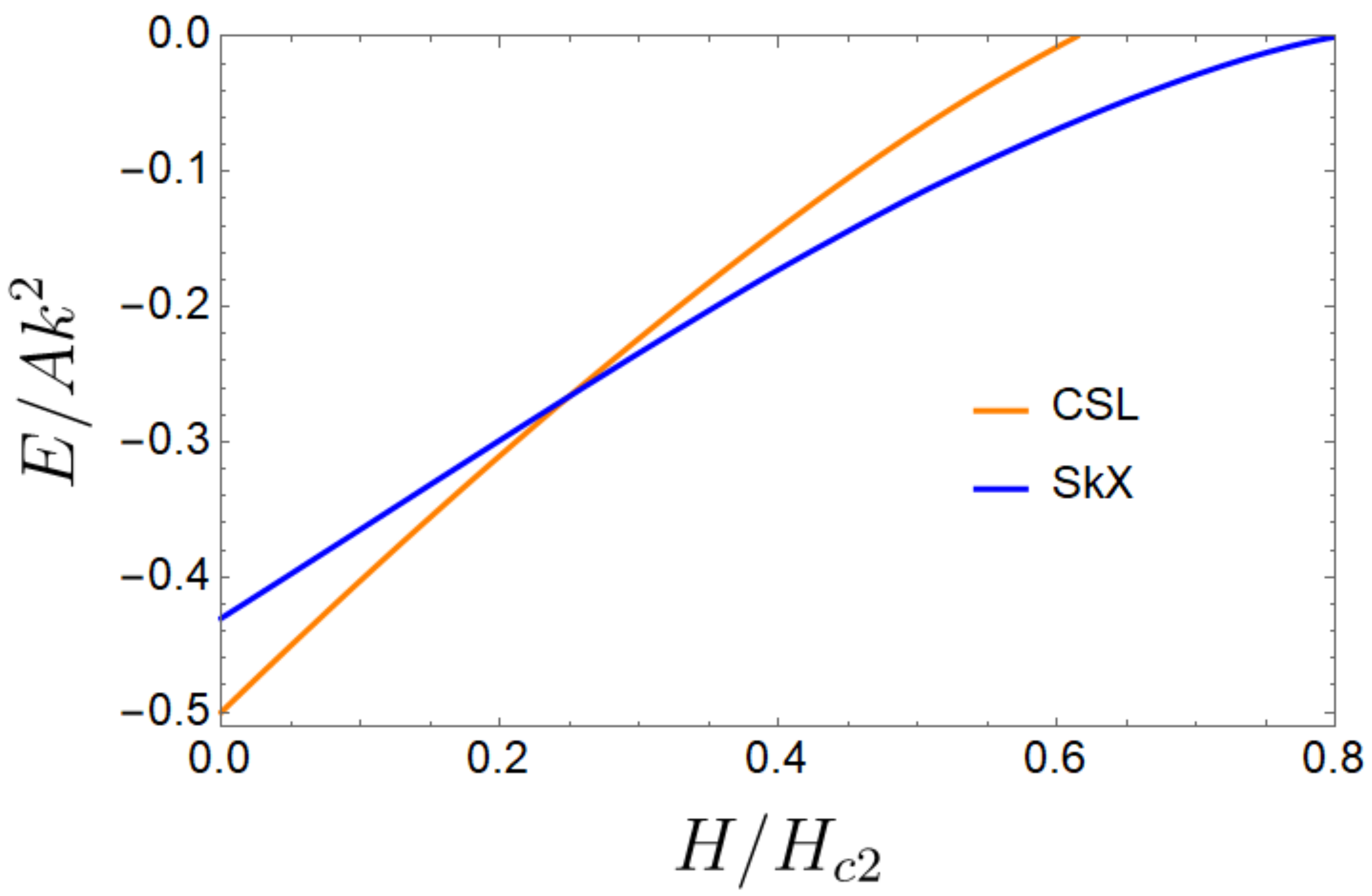
$$\mathscr{E} = \frac{1}{2}(\nabla \mathbf{S})^2 + \mathbf{S}\cdot\nabla\times\mathbf{S} - b\,S_z$$

Phase diagram at $T = 0$:

Simple helix $0 < b < 0.25$

Skyrmion phase $0.25 < b < 0.8$

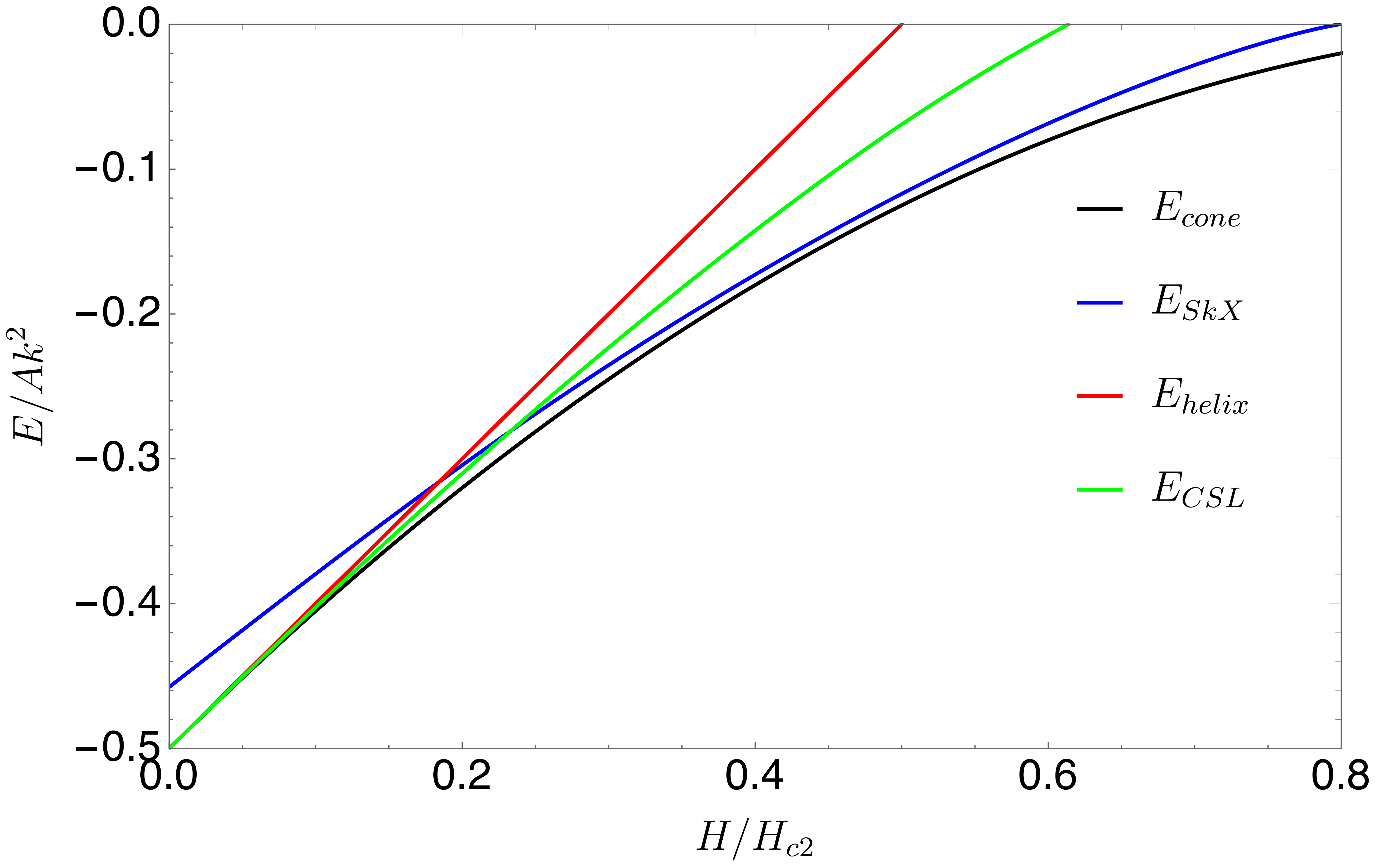
Uniform ferromagnet $b > 0.8$

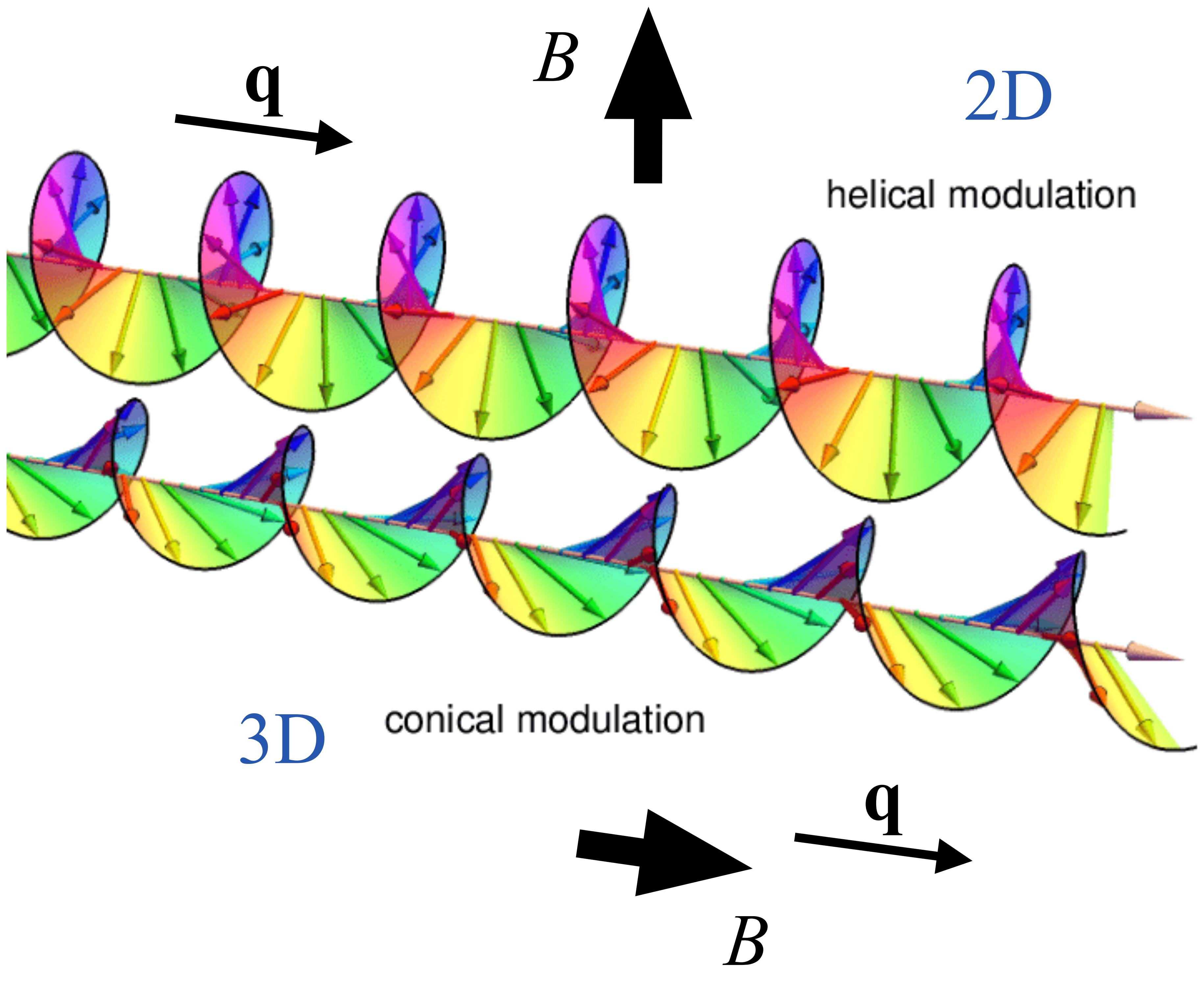


Belavin, Polyakov, JETP Letters (1975)
Bogdanov, Yablonsky, JETP (1989)

Difference between 2D and 3D

$$\mathscr{E} = \frac{1}{2}J(\nabla \mathbf{S})^2 + D\mathbf{S}\cdot\nabla\times\mathbf{S} - B\,S_z$$





researchgate.net

Stereographic projection

$$S^1 + iS^2 = \frac{2f}{1+f\bar{f}}, \quad S^3 = \frac{1-f\bar{f}}{1+f\bar{f}}$$

$\mathbf{S} = (0,0,1) \leftrightarrow f = 0$

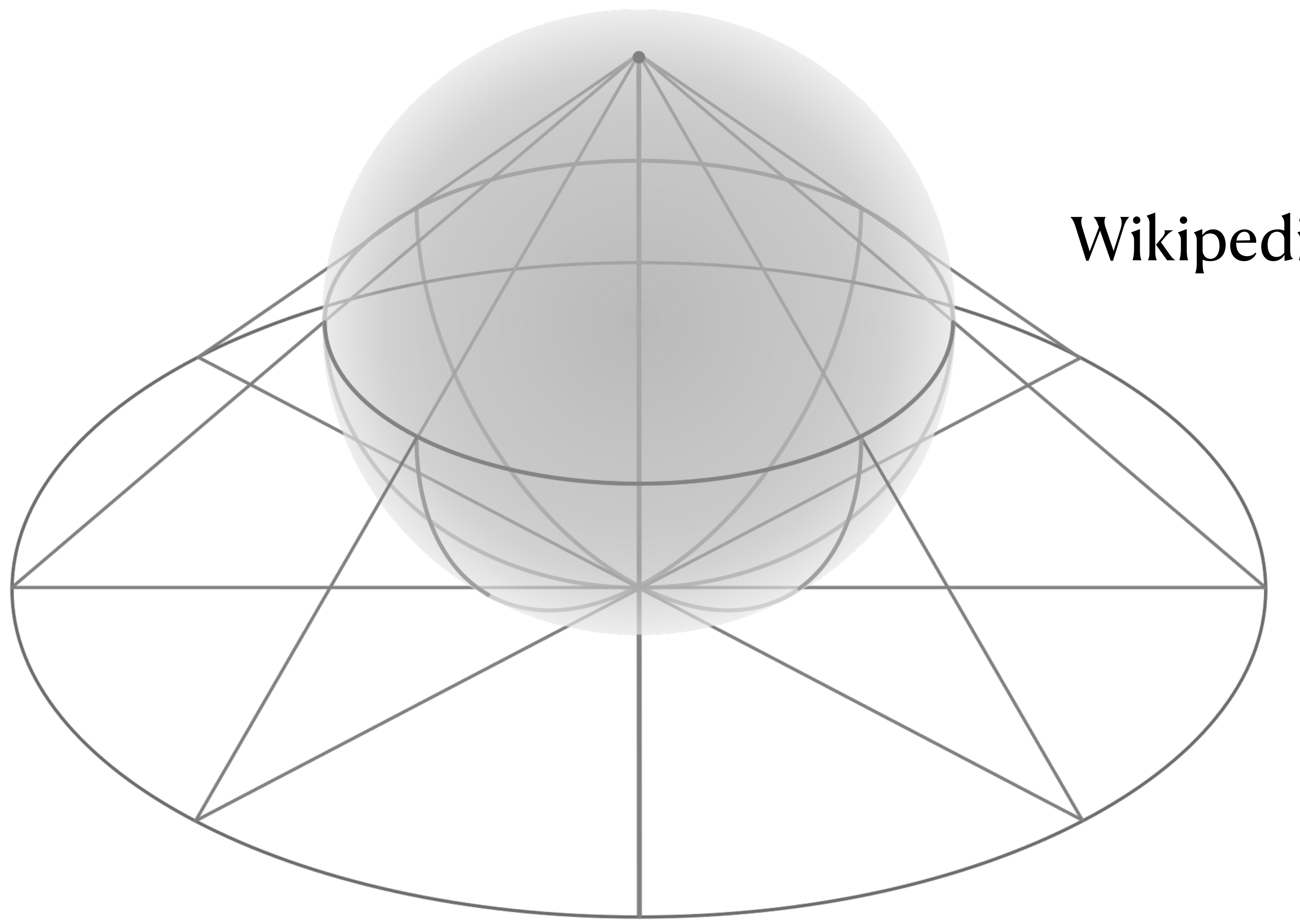
$\mathbf{S} = (0,0,-1) \leftrightarrow f = \infty$

$(z = x + iy, \bar{z} = x - iy)$

Topological charge:

$$Q = \frac{1}{4\pi}\int d^2\mathbf{r}\ \frac{4(\partial_z f \partial_{\bar{z}} \bar{f} - \partial_z \bar{f} \partial_{\bar{z}} f)}{(1+f\bar{f})^2}$$

$$Q = \frac{1}{4\pi}\int d^2\mathbf{r}\ \mathbf{S}\cdot(\partial_x \mathbf{S} \times \partial_y \mathbf{S})$$



Wikipedia

Energy and Lagrangian

$$L = \int d^2\mathbf{r}\,(\mathcal{T} - \mathcal{E}) \qquad \mathcal{T}[f] = \frac{i}{2}\frac{\bar{f}\partial_t f - f\partial_t \bar{f}}{1+f\bar{f}}$$

$$\mathcal{E} = \frac{4(\partial_z f \partial_{\bar{z}} \bar{f} + \partial_z \bar{f} \partial_{\bar{z}} f)}{(1+f\bar{f})^2} + \left\{ \frac{2i(\bar{f}^2\partial_{\bar{z}} f + \partial_{\bar{z}}\bar{f} - \partial_z f - f^2\partial_z \bar{f})}{(1+f\bar{f})^2} \right\} + \frac{2bf\bar{f}}{1+f\bar{f}}$$

Variation: $\delta L/\delta f = 0 \Rightarrow$

$$2f\partial_z \bar{f}\partial_{\bar{z}}\bar{f} - (1+f\bar{f})\partial_z\partial_{\bar{z}}\bar{f} - i\{\bar{f}\partial_{\bar{z}}\bar{f} + f\partial_z\bar{f}\} + \frac{1}{4}b\bar{f}(1+f\bar{f}) = 0$$

Any (anti)holomorphic f ⟷ Belavin,Polyakov (1975): $D = B = 0$

$$2f\partial_z \bar{f}\partial_{\bar{z}}\bar{f} - (1+f\bar{f})\partial_z\partial_{\bar{z}}\bar{f} = 0$$

See also: Metlov, PRB (2013)

Ansatz for skyrmion crystal

Belavin, Polyakov: $f = \sum z_j/(\bar{z} - Z_j)$

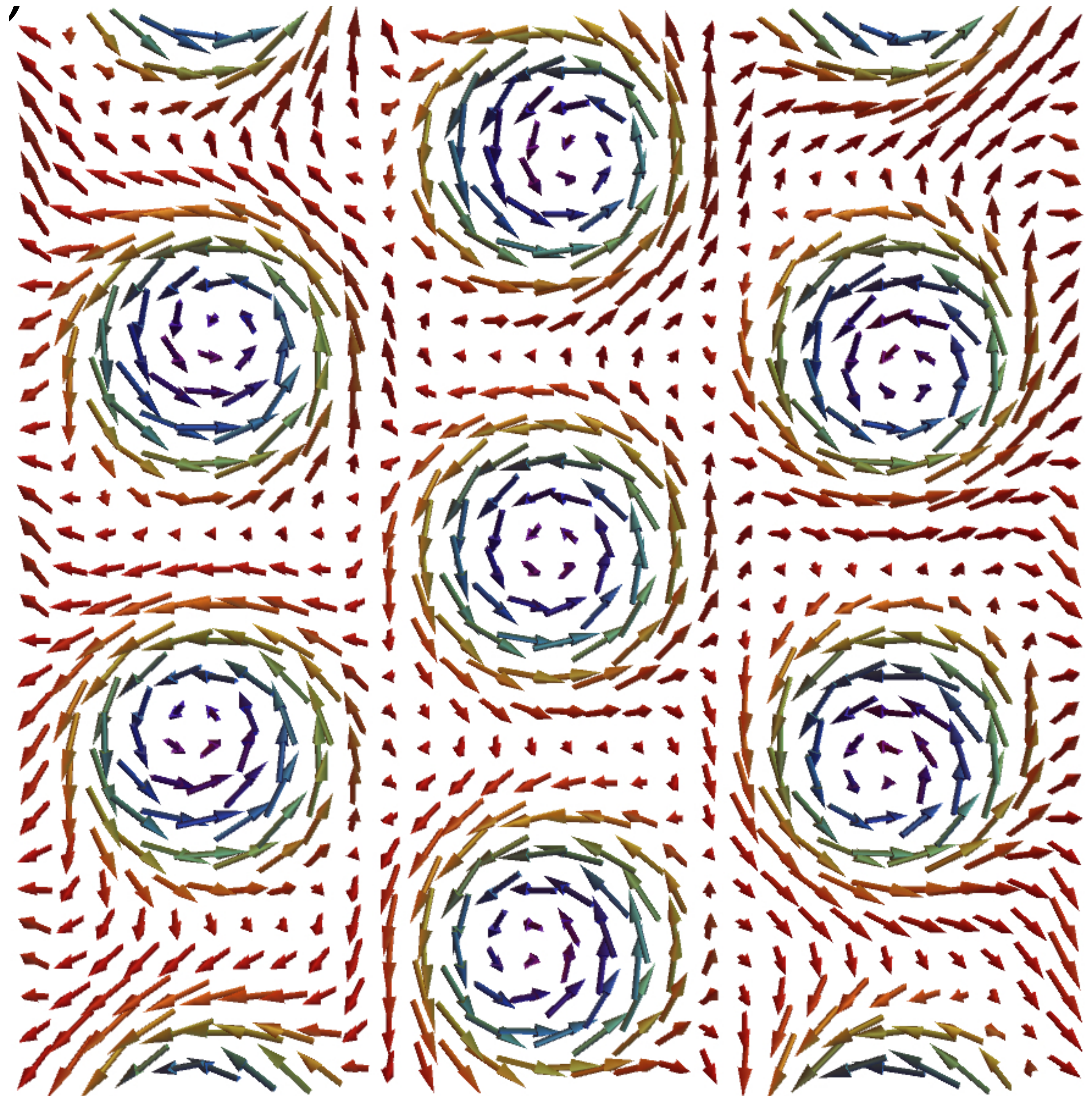
size z_j, position Z_j

$(z = x + iy, \bar{z} = x - iy)$

Now $D \neq 0, B \neq 0$

$$f_{SkX}(a, z_0) = \sum_{n,m} f_1(\mathbf{r} - n\mathbf{a}_1 - m\mathbf{a}_2)$$

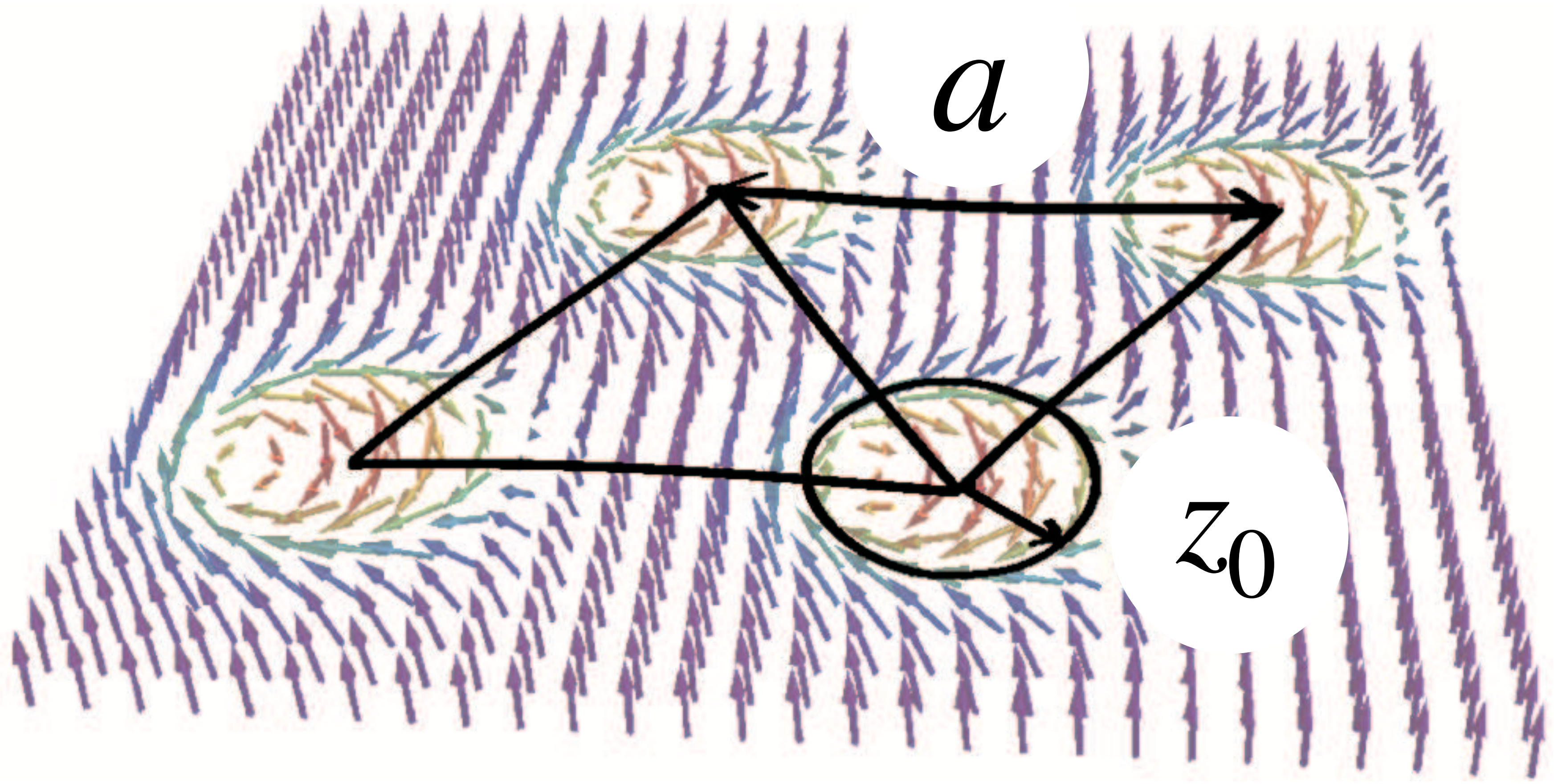
$$f_1 = \frac{i\, z_0\, \kappa(z\bar{z}/z_0^2)}{\bar{z}}$$



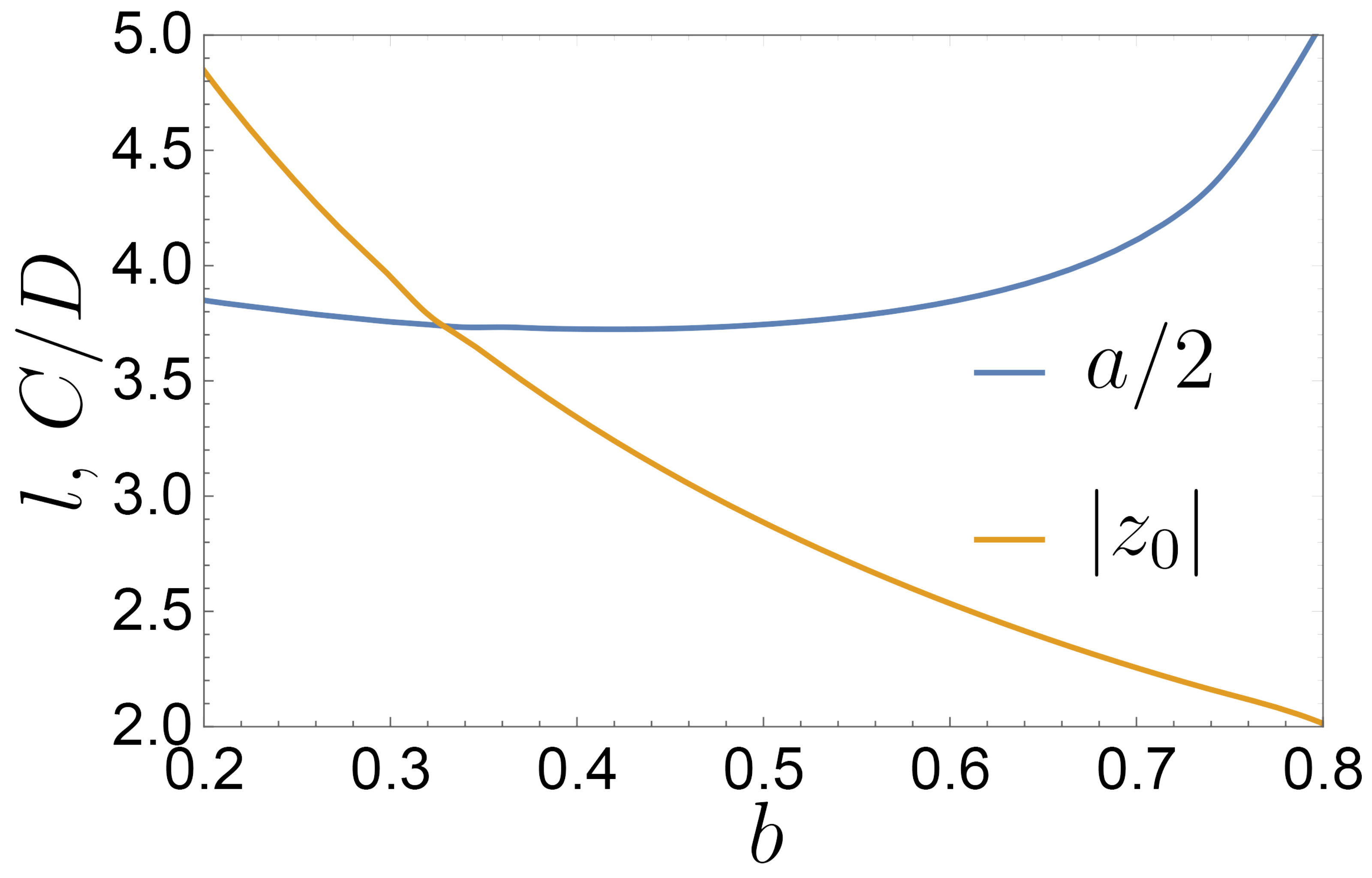
Timofeev, Sorokin, Aristov, JETP Letters (2019)

Timofeev, Sorokin, Aristov, PRB (2021)

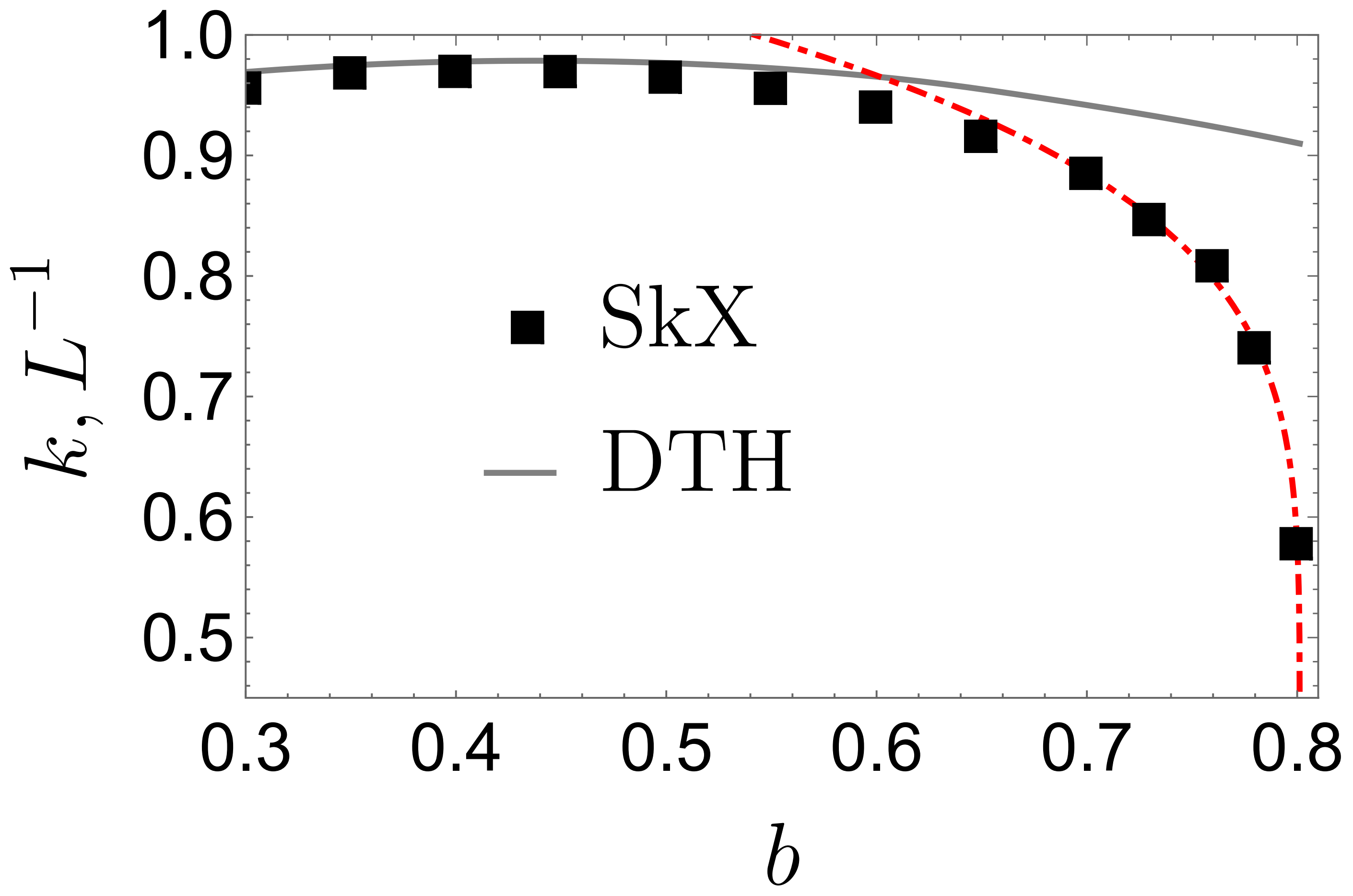
Shape and size of skyrmions



$$f_1 = \frac{i\, z_0\, \kappa(z\bar{z}/z_0^2)}{\bar{z}}, \quad \kappa(r^2) \simeq \exp(-r^2)$$



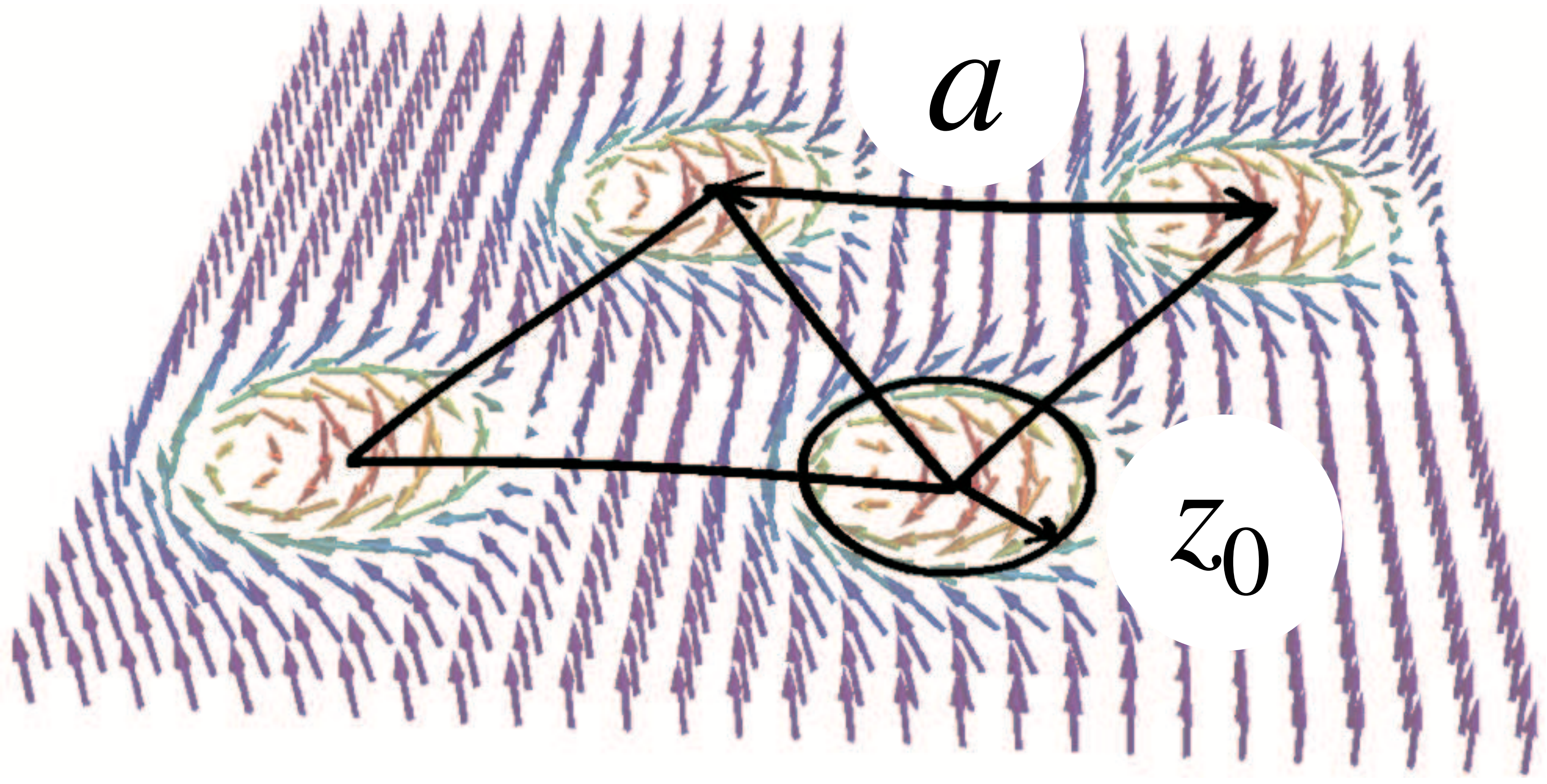
Skyrmion size



Inverse distance between skyrmions $k = 2\pi/a$

Timofeev, Sorokin, Aristov, JETP Letters (2019)
Timofeev, Sorokin, Aristov, PRB (2021)

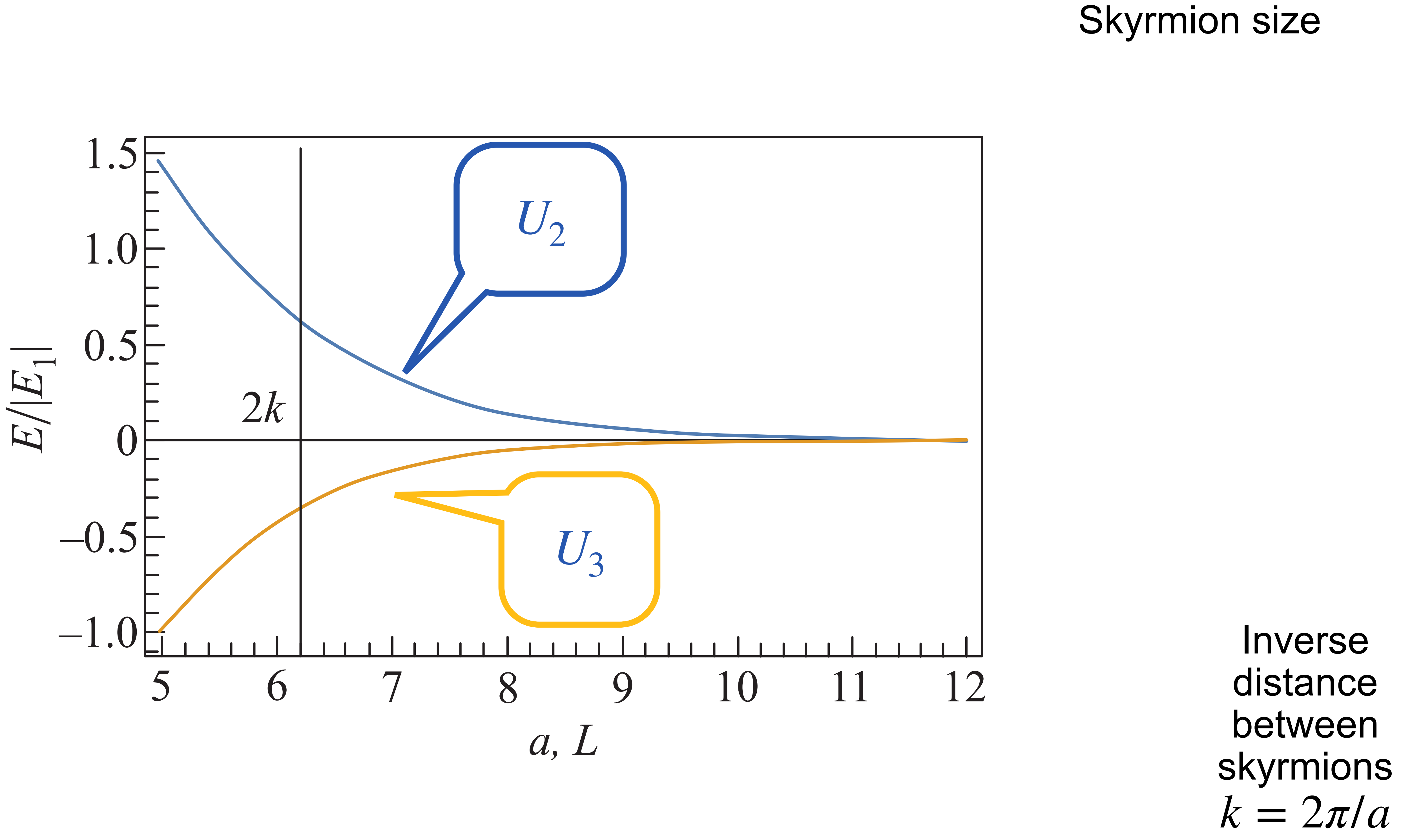
Interaction between skyrmions



$$U_2(z_0, a) = \mathcal{H}\left[f_1 + f_2\right] - \mathcal{H}[f_1] - \mathcal{H}[f_2]$$

$$U_3(a) = \mathcal{H}\left[\sum_{j=1,\ldots 4} f_j\right] - 4\mathcal{H}[f_1] - 5U_2(a)$$

Timofeev, Sorokin, Aristov, JETP Letters (2019)



Semiclassical method

$$f(t, z, \bar{z}) = f_0(z, \bar{z}) + \delta f(t, z, \bar{z})$$

$$\mathscr{L}[f_0 + \delta f] = \mathscr{L}[f_0] + \delta f \, \mathscr{L}_1[f_0] + \frac{1}{2}\delta f \, \delta f \, \mathscr{L}_2[f_0] + \ldots$$

Linear spin-wave theory

Overall translation $\mathbf{R}(t)$ = «Zero mode»

$$f(\mathbf{r}) = f_0 + (1 + f_0 \bar{f}_0)\, \psi(\mathbf{r} - \mathbf{R}(t))$$

$$\mathscr{L} = \frac{1}{2}(\bar{\psi}, \quad \psi)\left(-i\begin{pmatrix} \partial_t & 0 \\ 0 & -\partial_t \end{pmatrix} - \hat{\mathscr{H}}\right)\begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix}$$

Equations of motion

$$\hat{\mathscr{H}} = \begin{pmatrix} (-i\nabla + \mathbf{A})^2 + U & V \\ V^* & (i\nabla + \mathbf{A})^2 + U \end{pmatrix}$$

``Bogoliubov-de Gennes"

Any *f* providing extremum to the action

$$U = -4\frac{\partial_z f \partial_{\bar z} \bar f + \partial_z \bar f \partial_{\bar z} f}{(1+f\bar f)^2} + b\frac{1-f\bar f}{1+f\bar f} + \left\{ \frac{2i(f^2\partial_z \bar f + \partial_z f - \partial_{\bar z} \bar f - \bar f^2 \partial_{\bar z} f + 2if\bar f)}{(1+f\bar f)^2} \right\}$$

$$V = 8\frac{\partial_z f \partial_{\bar z} f(1-2f\bar f) + f(1+f\bar f)\partial_z \partial_{\bar z} f}{(1+f\bar f)^2} - \left\{ \frac{4i(3f^2\partial_z f - \partial_{\bar z} f(1-2f\bar f))}{(1+f\bar f)^2} \right\} - b\frac{2f^2}{1+f\bar f}$$

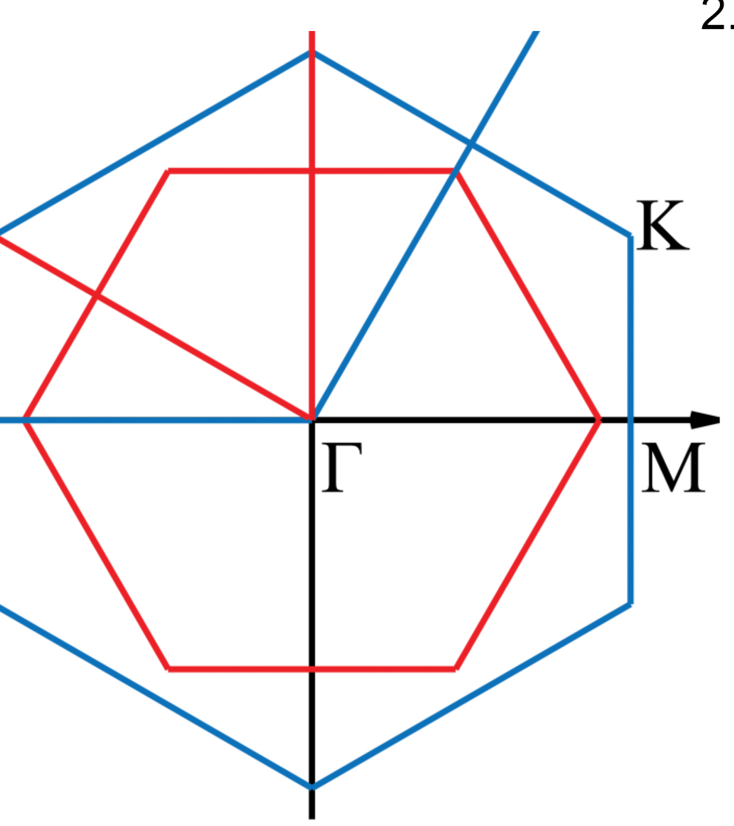
$$A_x = \frac{if\partial_x \bar f - i\bar f \partial_x f}{1+f\bar f} + \left\{ \frac{4\,\mathrm{Re} f}{1+f\bar f} \right\}$$

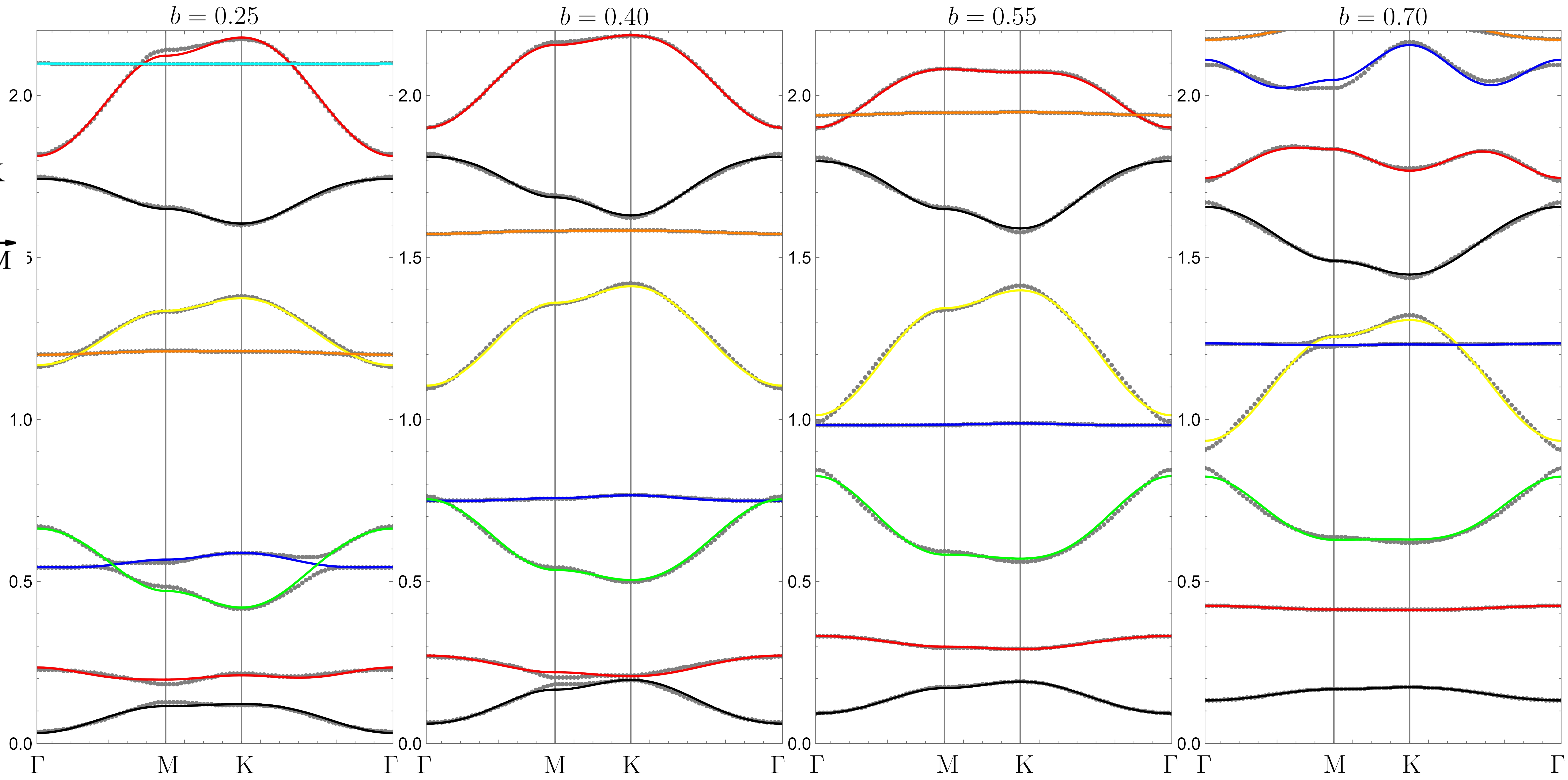
Gauge vector potential

Bogoliubov spinor

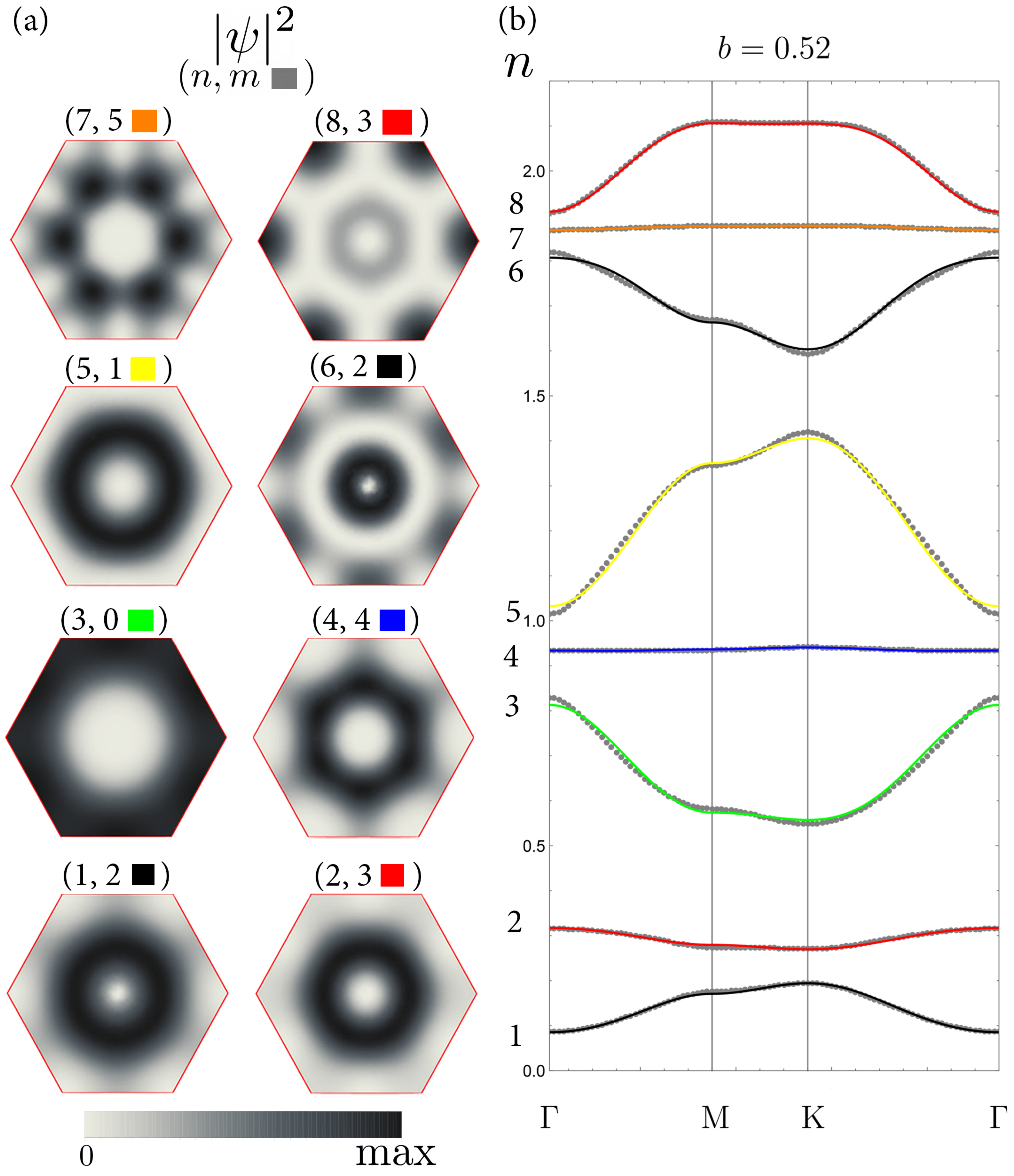
$$\left(\epsilon_n \sigma_3 - \hat{\mathscr{H}}\right)\begin{pmatrix} u_n \\ v_n \end{pmatrix} = 0$$

Spectrum: evolution with B





Spectrum: types of deformation



* Bogoliubov u-v spinors, most weight in the upper (u) component
* Bloch function strongly varying in the unit cell
* behavior at centers of the skyrmions, $\psi \sim \exp im\phi$

Deformations of skyrmions:

m=0 counterclockwise rotation
m=1 breathing mode
m=2 clockwise rotation, «zero mode»
m=3 elliptical deformation
m=4 triangular deformation, etc.

Timofeev, Aristov, PRB (2022)

Visualization of excitations

Dynamic susceptibility tensor

$$\chi_{ij}(\mathbf{k},t) = -i\theta(t)\langle[S_i(\mathbf{k},t), S_j(-\mathbf{k},0)]\rangle$$

$$S_i(\mathbf{k},t) = S_i^{(0)}(\mathbf{k}) + \sqrt{\frac{S}{2}}\sum_n (A_n^i(\mathbf{k})e^{-i\epsilon_n t}c_n + H.c.)$$

$$A_n^j(\mathbf{k}) = \int d\mathbf{r}\, e^{i\mathbf{kr}}(\bar{F}_j u_n + F_j v_n)$$

$$\chi_{ij}(\mathbf{k},\omega) = \frac{S}{2}\sum_n \left(\frac{\bar{A}_n^i(\mathbf{k})A_n^j(-\mathbf{k})}{\omega+\epsilon_n+i\delta} - \frac{A_n^i(\mathbf{k})\bar{A}_n^j(-\mathbf{k})}{\omega-\epsilon_n+i\delta} \right)$$

Timofeev, Aristov, JETP Letters (2023)

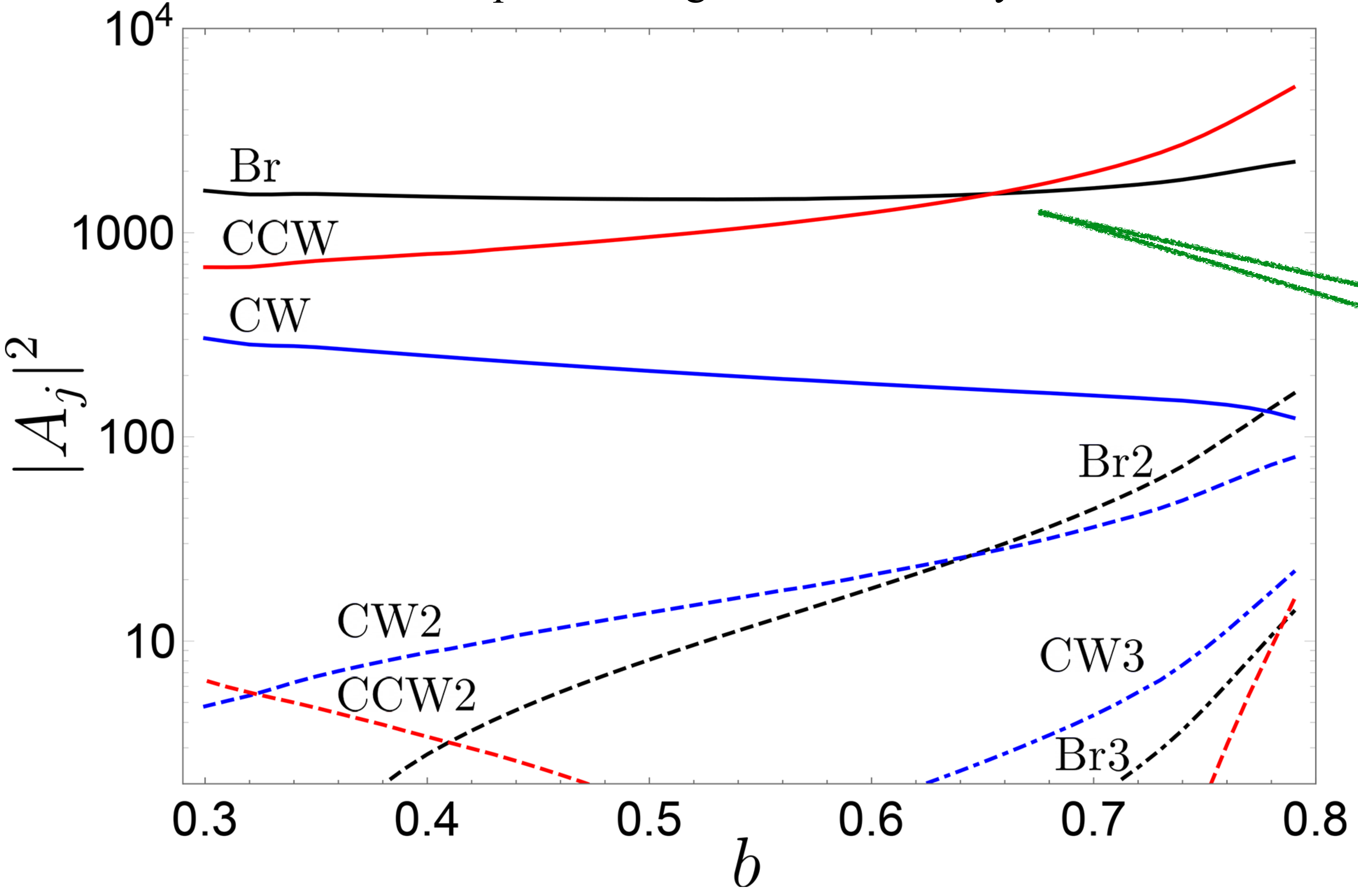
Uniform dynamic susceptibility

Lines crossing,
«Closing of the gap»

Timofeev, Aristov, JETP Letters (2023)

Uniform dynamic susceptibility

Spectral weight / Line intensity



Lines crossing,
«Closing of the gap»

Several resonance frequencies, in addition to three lowest ones

The lowest (gyrotropic) energy mode

Goldstone mode = equal displacements of all skyrmions

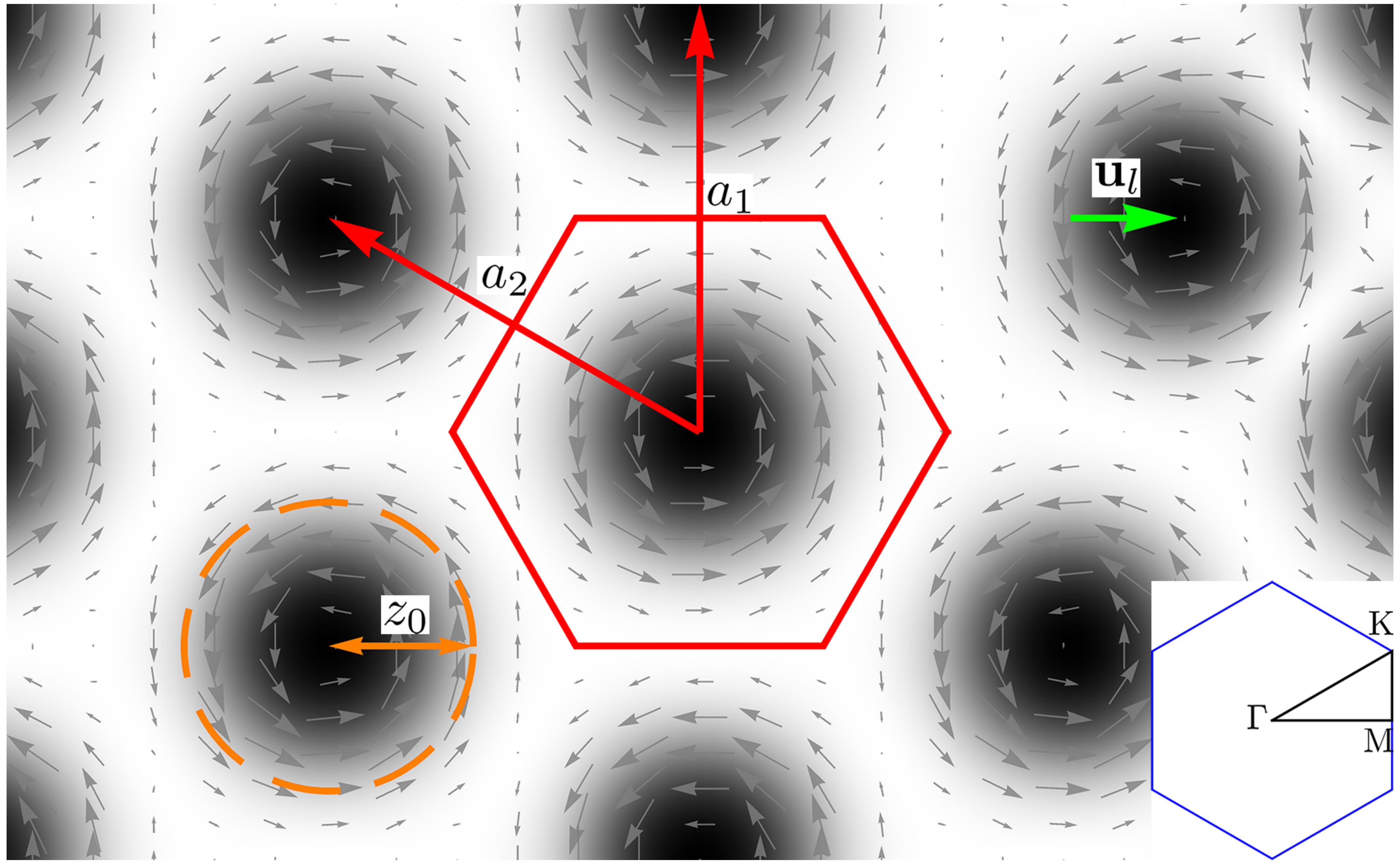
=>

Consider individual displacements

$$f_{SkX} = f_0 = \sum_{n,m} f_1(\mathbf{r} - n\mathbf{a}_1 - m\mathbf{a}_2)$$

$$f_{SkX} = \sum_{l} f_1(\mathbf{r} - \mathbf{r}_l^{(0)} + \mathbf{u}_l)$$

$$\mathbf{r}_l^{(0)} = n\mathbf{a}_1 + m\mathbf{a}_2$$



$$f_{SkX} \simeq f_0 + \sum_{l} \mathbf{u}_l \nabla f_1(\mathbf{r} - \mathbf{r}_l^{(0)})$$

Gyrotropic energy mode

$$\mathscr{L} = \frac{1}{2}(\bar{\psi}, \quad \psi)\left(-i\begin{pmatrix}\partial_t & 0 \\ 0 & -\partial_t\end{pmatrix} - \hat{\mathscr{H}}\right)\begin{pmatrix}\psi \\ \bar{\psi}\end{pmatrix}$$

$$\hat{\mathscr{H}} = \begin{pmatrix}(-i\nabla + \mathbf{A})^2 + U & V \\ V^* & (i\nabla + \mathbf{A})^2 + U\end{pmatrix}$$

$$\sum_l \mathbf{u}_l \nabla f_1(\mathbf{r} - \mathbf{r}_l^{(0)}) = (1 + f_0\bar{f}_0)\,\psi(\mathbf{r})$$

$$\begin{pmatrix}\psi \\ \bar{\psi}\end{pmatrix} = \frac{1}{1 + f_0\bar{f}_0}\sum_j \begin{pmatrix}\partial_{\bar{z}}f_j, \partial_z f_j \\ \partial_{\bar{z}}\bar{f}_j, \partial_z\bar{f}_j\end{pmatrix}\begin{pmatrix}u_j^- \\ u_j^+\end{pmatrix}$$

$$\equiv \sum_j \mathscr{O}_j \begin{pmatrix}u_j^- \\ u_j^+\end{pmatrix}$$

$$u_j^\pm = u_j^x \pm i u_j^y$$

$$\mathscr{L} = \frac{1}{2}\sum_{lj}(u_l^+, u_l^-)\left(-i\hat{\mathscr{K}}_{lj}\partial_t - \hat{\mathscr{H}}_{lj}\right)\begin{pmatrix}u_j^- \\ u_j^+\end{pmatrix}$$

$$\hat{\mathscr{K}}_{lj} = \int d\mathbf{r}\, \mathscr{O}_l^\dagger . \sigma_3 . \mathscr{O}_j$$

$$\hat{\mathscr{H}}_{lj} = \int d\mathbf{r}\, \mathscr{O}_l^\dagger . \begin{pmatrix}(-i\nabla + \mathbf{A})^2 + U & V \\ V^* & (i\nabla + \mathbf{A})^2 + U\end{pmatrix} . \mathscr{O}_j$$

Gyrotropic energy mode

$$\mathcal{L} = \frac{1}{2} \sum_{lj} \left(u_l^+, \quad u_l^-\right) \left(-i\hat{\mathcal{K}}_{lj}\partial_t - \hat{\mathcal{H}}_{lj}\right) \begin{pmatrix} u_j^- \\ u_j^+ \end{pmatrix}$$

- $\hat{\mathcal{K}}_{lj}, \hat{\mathcal{H}}_{lj}$ depend only on $\mathbf{r}_l^{(0)} - \mathbf{r}_j^{(0)}$
- Expect the property $\sum_j \hat{\mathcal{H}}_{lj} = \sum_l \hat{\mathcal{H}}_{lj} = 0$
- $\hat{\mathcal{K}}_{lj}, \hat{\mathcal{H}}_{lj}$ decrease rapidly with distance
- Thiele equation: $u_j^\pm = 0$ for all $j \neq l$: $\mathcal{K}_{ll} u_l^x \dot{u}_l^y - h_1((u_l^x)^2 + (u_l^y)^2)$

$$u_j^x \pm i u_j^y = \sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}_j} u_{\mathbf{q}}^\pm$$

$$\mathcal{L} = \frac{1}{2} \sum_{\mathbf{q}} \left(u_{-\mathbf{q}}^+, u_{-\mathbf{q}}^-\right) \left(-i\hat{\mathcal{K}}_{\mathbf{q}}\partial_t - \hat{\mathcal{H}}_{\mathbf{q}}\right) \begin{pmatrix} u_{\mathbf{q}}^- \\ u_{\mathbf{q}}^+ \end{pmatrix}$$

$$\hat{\mathcal{K}}_{\mathbf{q}} = (\pi + k_1 \gamma_s(\mathbf{q}))\sigma_3$$

$$\hat{\mathcal{H}}_{\mathbf{q}} = \begin{pmatrix} h_1\gamma_s(\mathbf{q}), h_2\,\gamma_d^*(\mathbf{q}) \\ h_2\,\gamma_d(\mathbf{q}), h_1\gamma_s(\mathbf{q}) \end{pmatrix}$$

Gyrotropic energy mode

$$\mathscr{L} = \frac{1}{2}\sum_{\mathbf{q}} \left(u^+_{-\mathbf{q}}, u^-_{-\mathbf{q}}\right)\left(-i\hat{\mathscr{K}}_{\mathbf{q}}\partial_t - \hat{\mathscr{H}}_{\mathbf{q}}\right)\begin{pmatrix} u^-_{\mathbf{q}} \\ u^+_{\mathbf{q}} \end{pmatrix}$$

$$\hat{\mathscr{K}}_{\mathbf{q}} = (\pi + k_1\gamma_s(\mathbf{q}))\sigma_3$$

$$\hat{\mathscr{H}}_{\mathbf{q}} = \begin{pmatrix} h_1\gamma_s(\mathbf{q}), h_2\,\gamma_d^*(\mathbf{q}) \\ h_2\,\gamma_d(\mathbf{q}), h_1\gamma_s(\mathbf{q}) \end{pmatrix}$$

Equation of motion

$$u^{\pm}_{\mathbf{q}}(t) = e^{i\omega t}u^{\pm}_{\mathbf{q}}$$

$$\det(\omega\hat{\mathscr{K}}_{\mathbf{q}} - \hat{\mathscr{H}}_{\mathbf{q}}) = 0$$

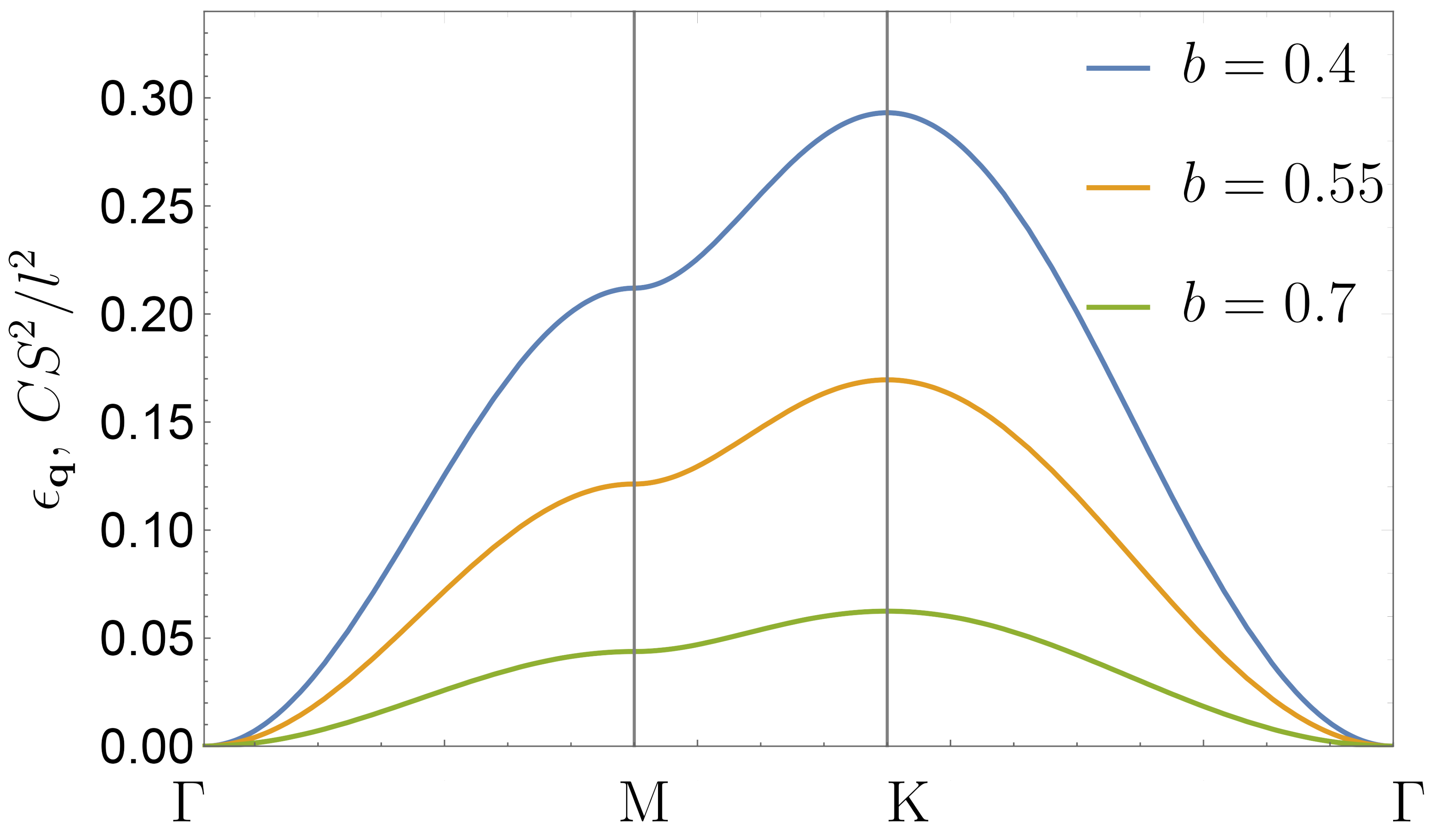
Dispersion relation

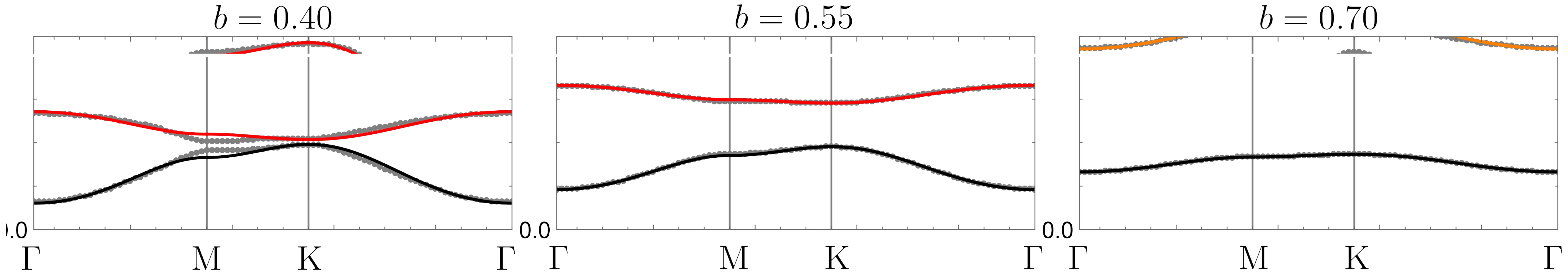
$$\epsilon_{\mathbf{q}} = \frac{(h_1^2\gamma_s^2(\mathbf{q}) - h_2^2\,|\,\gamma_d(\mathbf{q})\,|^2)^{1/2}}{\pi + k_1\gamma_s(\mathbf{q})}$$

$$\gamma_s(\mathbf{q}) = \sum_{\mathbf{d}} e^{-i\mathbf{q}\mathbf{d}} - 6 = 2\left(2\cos\frac{\sqrt{3}}{2}q_x a\cos\frac{1}{2}q_y a + \cos q_y a - 3\right)$$

$$\gamma_d(\mathbf{q}) = \sum_{\mathbf{d}} e^{-i\mathbf{q}\mathbf{d}}e^{2i\phi_d} = 2\left(\cos\frac{\sqrt{3}}{2}q_x a\cos\frac{1}{2}q_y a - \cos q_y a - i\sqrt{3}\sin\frac{\sqrt{3}}{2}q_x a\sin\frac{1}{2}q_y a\right)$$

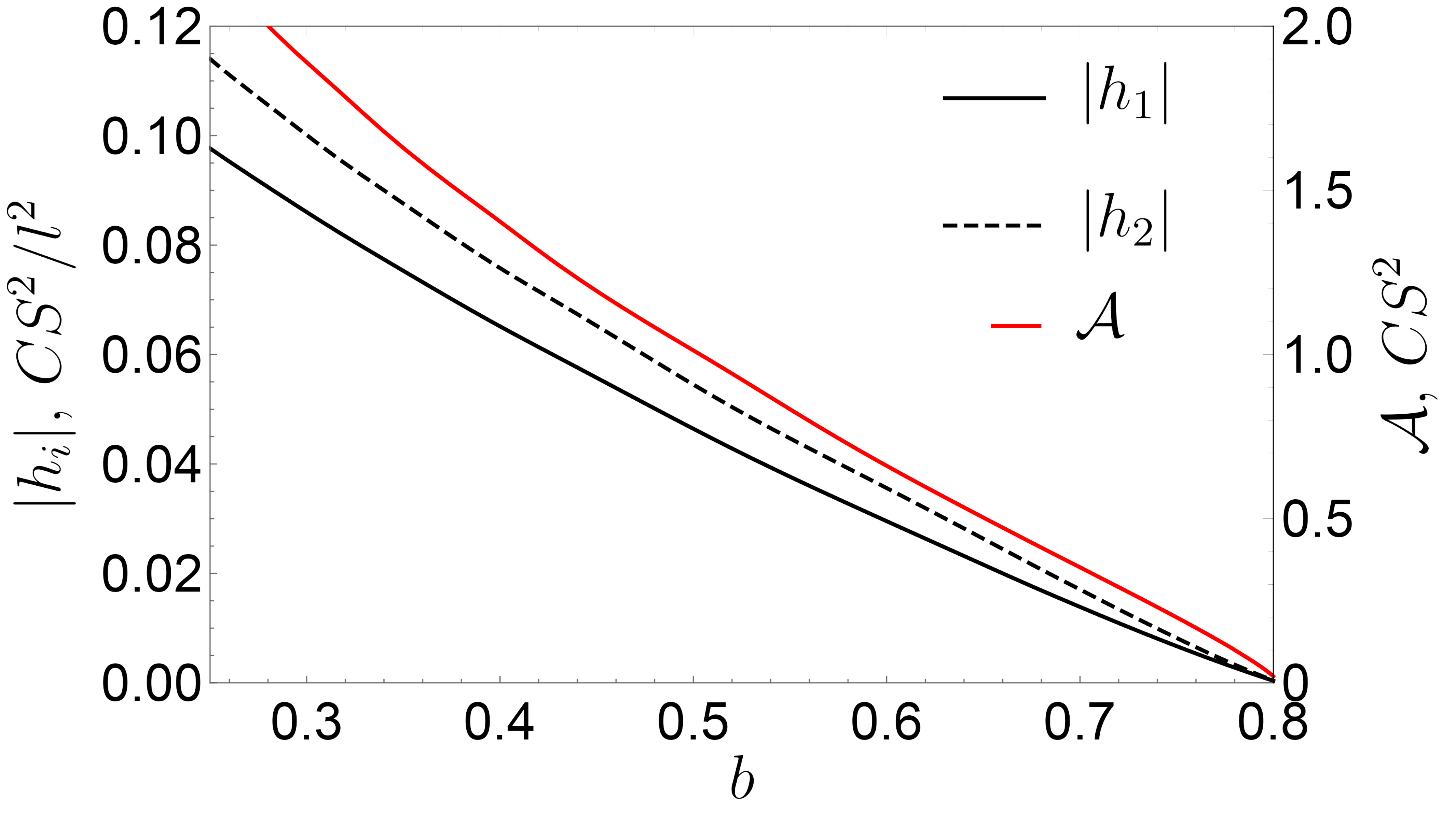
Gyrotropic energy mode





$$\epsilon_{\mathbf{q}} = \frac{(h_1^2\gamma_s^2(\mathbf{q}) - h_2^2|\gamma_d(\mathbf{q})|^2)^{1/2}}{\pi + k_1\gamma_s(\mathbf{q})}$$

$$\epsilon_{\mathbf{q}} \simeq \frac{3}{4\pi}q^2a^2\sqrt{4h_1^2 - h_2^2} \equiv \mathscr{A}q^2$$



Gyrotropic energy mode, Green's function

$$\mathscr{L} = \frac{1}{2}\sum_{\mathbf{q}} \left(u^{\parallel}_{-\mathbf{q}}, u^{\perp}_{-\mathbf{q}}\right) \begin{pmatrix} -A_{\parallel}q^2, -2\pi\partial_t \\ 2\pi\partial_t, -A_{\perp}q^2 \end{pmatrix} \begin{pmatrix} u^{\parallel}_{\mathbf{q}} \\ u^{\perp}_{\mathbf{q}} \end{pmatrix}$$

$$A_{\parallel} = -\tfrac{3}{2}(2h_1 + h_2)a^2\,, \quad A_{\perp} = -\tfrac{3}{2}(2h_1 - h_2)a^2$$

$$u^{\parallel}_{\mathbf{q}} = \frac{1}{\sqrt{4\pi\varkappa}}(c^{\dagger}_{\mathbf{q}}e^{i\epsilon_{\mathbf{q}}t} + c_{-\mathbf{q}}e^{-i\epsilon_{\mathbf{q}}t})$$

$$u^{\perp}_{\mathbf{q}} = i\frac{\sqrt{\varkappa}}{\sqrt{4\pi}}(c^{\dagger}_{\mathbf{q}}e^{i\epsilon_{\mathbf{q}}t} - c_{-\mathbf{q}}e^{-i\epsilon_{\mathbf{q}}t})$$

$$\varkappa = \sqrt{A_{\parallel}/A_{\perp}} \simeq 1.98$$

For phonons: longitudinal and transverse sound modes

Now $u^{\perp}_{-\mathbf{q}}$ is canonically conjugate to $u^{\parallel}_{\mathbf{q}}$

Second quantization: $[u^{\parallel}_{\mathbf{q}}, 2\pi\, u^{\perp}_{-\mathbf{q}}] = i\hbar$

Only one sort of bosons (would be two for phonons)

Gyrotropic energy mode, Green's function

$$u^{\parallel}_{\mathbf{q}} = \frac{1}{\sqrt{4\pi\varkappa}}(c^{\dagger}_{\mathbf{q}} e^{i\epsilon_{\mathbf{q}} t} + c_{-\mathbf{q}} e^{-i\epsilon_{\mathbf{q}} t})$$

$$u^{\perp}_{\mathbf{q}} = i\frac{\sqrt{\varkappa}}{\sqrt{4\pi}}(c^{\dagger}_{\mathbf{q}} e^{i\epsilon_{\mathbf{q}} t} - c_{-\mathbf{q}} e^{-i\epsilon_{\mathbf{q}} t})$$

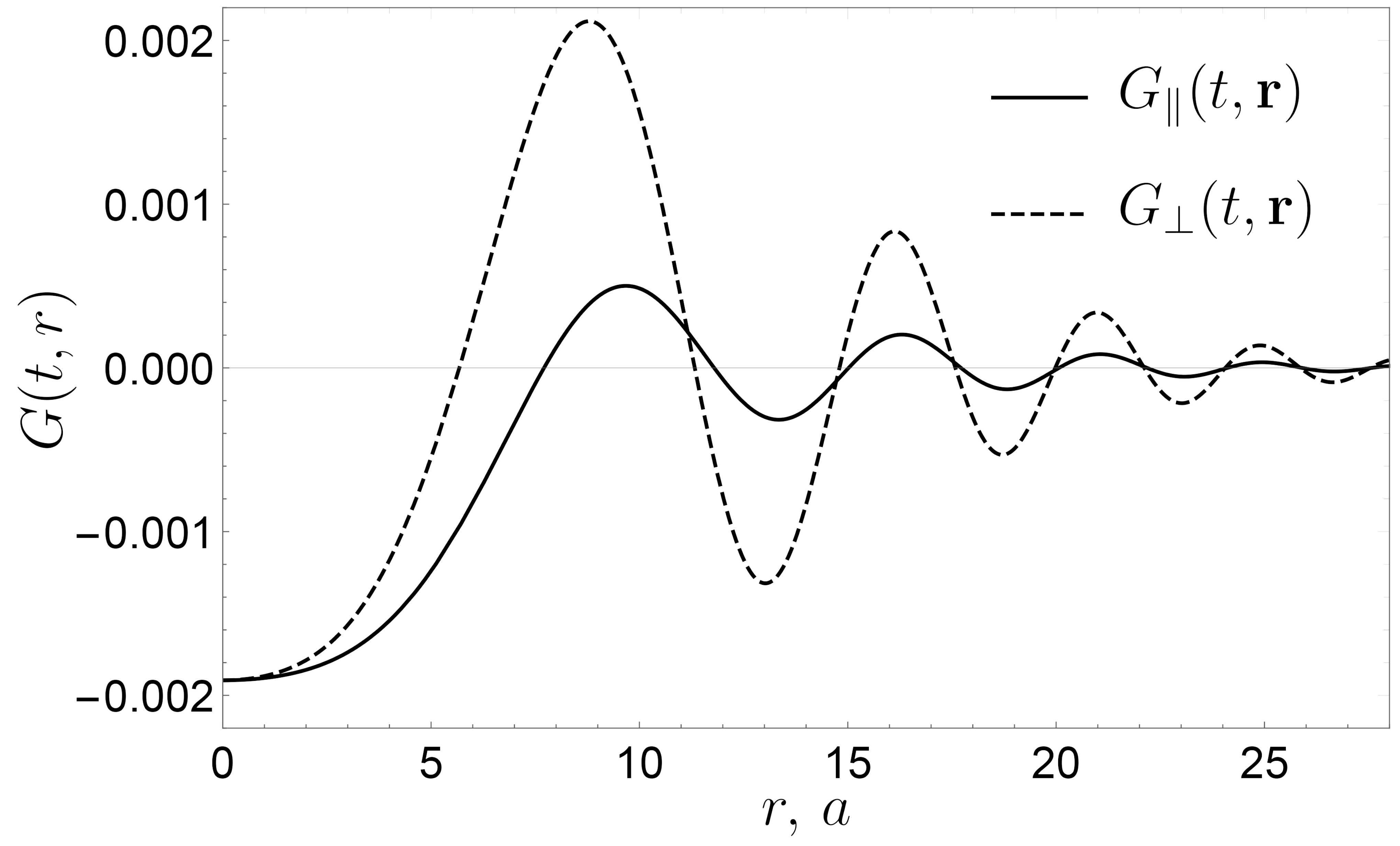
$$G(t, \mathbf{q}) = -i\vartheta(t)\begin{pmatrix} [u^x_{-\mathbf{q}}(t), u^x_{\mathbf{q}}], & [u^x_{-\mathbf{q}}(t), u^y_{\mathbf{q}}] \\ [u^y_{-\mathbf{q}}(t), u^x_{\mathbf{q}}], & [u^y_{-\mathbf{q}}(t), u^y_{\mathbf{q}}] \end{pmatrix}$$

$$\begin{pmatrix} u^x_{\mathbf{q}} \\ u^y_{\mathbf{q}} \end{pmatrix} = \begin{pmatrix} \cos\phi_q, & -\sin\phi_q \\ \sin\phi_q, & \cos\phi_q \end{pmatrix} \cdot \begin{pmatrix} u^{\parallel}_{\mathbf{q}} \\ u^{\perp}_{\mathbf{q}} \end{pmatrix}$$

For $t\mathscr{A} \gg ra, r \gg a$

$$G(t, \mathbf{r}) = \frac{\sqrt{3}a^2\rho}{4\pi^2 r^2}\left[-\cos(\rho)\frac{\varkappa + \varkappa^{-1}}{2}\begin{pmatrix} 1,0 \\ 0,1 \end{pmatrix} + \sin(\rho)\begin{pmatrix} 0,1 \\ -1,0 \end{pmatrix} + F(\rho)\frac{\varkappa - \varkappa^{-1}}{2}\begin{pmatrix} \cos 2\phi, \sin 2\phi \\ \sin 2\phi, -\cos 2\phi \end{pmatrix}\right]$$

$$F(z) = \cos z - \sin z / z, \qquad \rho = \frac{r^2}{4t\mathscr{A}}$$



Заключения и перспективы

- Скирмионное основное состояние магнетиков без центра инверсии обладает «топологическим зарядом». Удобно представлять такое состояние суммой образов* отдельных скирмионов с единичным зарядом (* в методе стереографической проекции).
- Метод стер.про. надежно определяет 1) энергию основного состояния и 2) спектр возбуждений скирмионного кристалла (СкК)
- Построена эффективная теория для некоторых низколежащих возбуждений СкК
- Показан топологический переход в спектре магнонов СкК

- Плавление скирмионной решетки ?
- Аномалии в холловской теплопроводности СкК ?
- Краевые магнонные состояния внутри СкК ?
- Магнетоупругий резонанс ?