

# DM HELICES AND MAGNETIC FIELD

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PNPI

# SPIN HELIX

$$\mathbf{S}_{\mathbf{R}} = S[(\mathbf{A}e^{i\mathbf{k}\cdot\mathbf{R}} + \mathbf{A}^*e^{-i\mathbf{k}\cdot\mathbf{R}}) \cos \alpha + \hat{c} \sin \alpha],$$

$$\mathbf{A} = (\hat{a} - i\hat{b})/2, \quad \hat{a}^2 = \hat{b}^2 = 1, \quad (\hat{a} \cdot \hat{b}) = 0,$$

$$\hat{c} = [\hat{a} \times \hat{b}], \quad \alpha \rightarrow \text{cone angle}..$$

# FELIX STRUCTURE

Helical structure: Wave vector  $\mathbf{k}$

$\hat{c}$  is perpendicular to spin rotation

Two main cases:

$\hat{c} \parallel \mathbf{k}$  B20 magnets.

$\hat{c} \perp \mathbf{k}$  cycloid ( Multifractoiks). .

# DMI 1

Spins are rotated by DMI:

$$V_{DM} = \mathbf{D} \cdot [\mathbf{S}_1 \times \mathbf{S}_2] = DS^2 \sin \theta_{12}.$$

DMI appears due to inversion symmetry

breacking in the center of the bond **b**

connecting two spins (Moroya theorem).

# DMI 2

Three breaking ways.

i.  $Mn - Si - Mn$  bond along  $\hat{x}$ .

$Si$  shifting  $||\hat{x} : L_{MnSi} < L_{Si-Mn}$ ,  $\mathbf{D} \sim [\hat{y} \times \hat{z}]||\hat{x}$ .

$\mathbf{k} \parallel \pm \hat{c}$ , Simple heliz.

ii.  $Mn - O - Mn$  bond.  $O$  shifting along  $y$  or  $z$ :

$\mathbf{D} \sim [\hat{x} \times \hat{y}]||\hat{z}$  or  $[\hat{x} \times \hat{z}]||\hat{y}$

$\mathbf{k} \perp \hat{c} \rightarrow$ Cycloids.

iii. Mirror symmetry breaking  $\hat{z} \neq -\hat{z}$

$\mathbf{D} \sim [\hat{z} \times \mathbf{b}]$ . Cycloid.

# ENERGY

Exchange and DM interactions  
+ magnetic field

$$H = (1/2) \sum [J_{\mathbf{R},\mathbf{R}'} (\mathbf{S}_{\mathbf{R}} \cdot \mathbf{S}_{\mathbf{R}'}) + (\mathbf{D}_{\mathbf{R},\mathbf{R}'} \cdot [\mathbf{S}_{\mathbf{R}} \times \mathbf{S}_{\mathbf{R}'}])] + \sum (\mathbf{H} \cdot \mathbf{S}_{\mathbf{R}}),$$

$\mathbf{D}_{\mathbf{R}',\mathbf{R}} = -\mathbf{D}_{\mathbf{R},\mathbf{R}'}.$

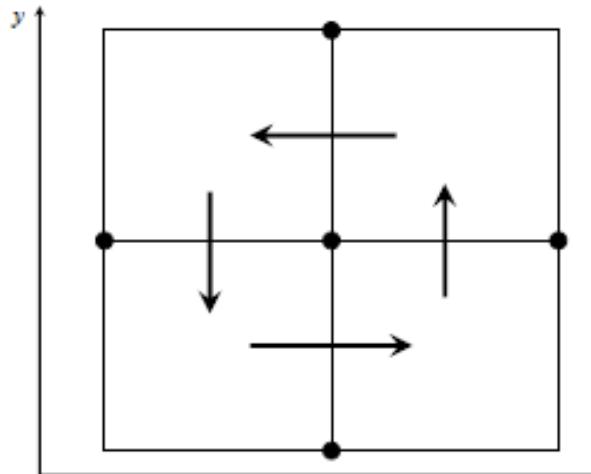
# HELIX ENERGY

$$E = S^2 \{ J_0 \sin^2 \alpha + [J_{\mathbf{k}} + i(\mathbf{D}_{\mathbf{k}} \cdot \hat{c})] \} \cos^2 \alpha + S(\mathbf{H} \cdot \hat{c}).$$

$$J_{\mathbf{k}} = \sum_{\mathbf{b}} J_{\mathbf{b}} \cos \mathbf{k} \cdot \mathbf{b},$$

$$\mathbf{D}_{\mathbf{k}} = i \sum_{\mathbf{b}} d_{\mathbf{b}} [\hat{z} \times \hat{b}] \sin \mathbf{k} \cdot \mathbf{b}$$

# SQUARE SURFACE LAYYR



Arrows are DM vectors

$$\mathbf{D}_x = d[\hat{y} \times \hat{z}], \quad \mathbf{D}_y = d[-\hat{x} \times \hat{z}]$$

$$J_{\mathbf{k}} = 2J(\cos k_x + \cos k_y),$$

$$\mathbf{D}_1 = 2id(-\sin k_x \sin k_y) \hat{z} \approx 2id[\hat{z} \times \mathbf{k}]$$

# AF LATTICE

$$\mathbf{k} \rightarrow \mathbf{k} + \pi(1, 1)$$

$$\tan k_x = k \hat{c}_y, ; \tan k_y = -k \hat{c}_x, \quad k = d/J$$

$$\sin \alpha = -\frac{(\mathbf{H} \cdot \hat{\mathbf{c}})}{H_c}, \quad H_c = \simeq 4SJ_0, \quad J_0 = 4J.$$

$H_c$ -spin flip field.

# In-Plane FIELD

$$E = -S^2 J_0 \left\{ 1 + \frac{k^2}{4} - \frac{k^4}{8} [\sin^4 \phi + \cos^4 \phi] - W \cos^2(\phi - \Phi) \right\},$$

$$\hat{c} = (\cos \phi, \sin \phi), \quad \mathbf{H} = H(\cos \Phi, \sin \Phi) \quad W = (H/4H_c k^2)^2.$$

TERMS: 1. AF energy. 2. Helix part.

3. DMI anisotropy. 4. Field energy.

# CHIRAL DOMAINS 1

$H = 0$  Energy minima along square

diagonals (easy  $\hat{c}$  directions).

Maxima along ridges (hard directions).

Four chiral domains:

$\mathbf{k} \perp \hat{c}$  CYCLOIDS.

$H \gg H_c k^2 \rightarrow \hat{c} \parallel \pm \mathbf{H}$  Two domains.

## CHIRAL DOMAINS 2

Strong field  $H > H_d = H_c k^2 / 2\sqrt{2}$ :  $\hat{c} \parallel \pm \mathbf{H}$ .

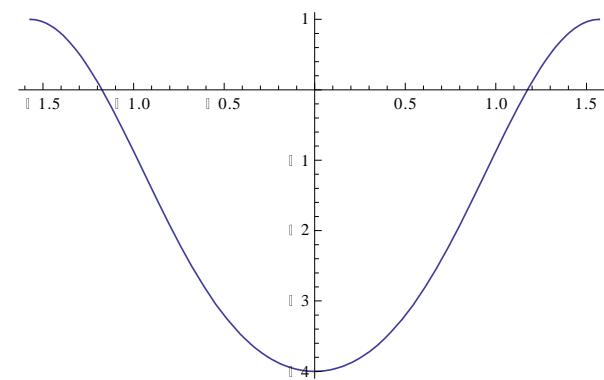
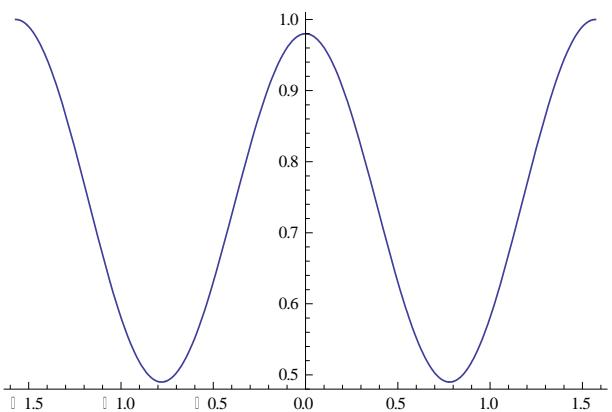
Weak field  $H < H_d$ .  $\mathbf{H} \perp (1,0)$ .  $\pm \pi/4$  domains rotate.

$\mathbf{H} \parallel (1,1)$ ,  $H < H_d$ . Both  $\pm \pi/4$  are in their places.

$H = H_d$ .  $-\pi/4$  domain jumps to plus place.

This state is stable below  $H_d$ .

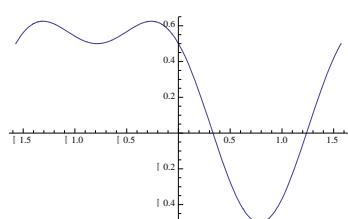
# FIELD ||(1,0)



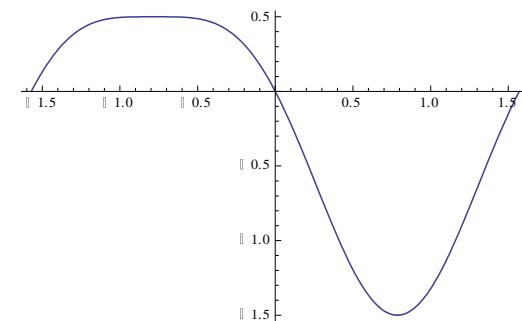
$H = 0$

$H \gg H_d$

FIELD || (1,1)



$$\mathcal{H} = H_d/\sqrt{2}$$



$$H = H_d$$

# Out-of--plane FIELD 1

$$(\mathbf{H} \cdot \hat{z}) = H \cos \Theta,$$

$$\hat{c} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta).$$

$$k_x = -k \sin \phi \sin \theta, \quad k_y = k \cos \phi \sin \theta. \quad .$$

$$E = -S^2 J_0(k^2/4) \sin^2 \theta - (SH^2/2H_c) \cos^2(\theta - \Theta).$$

$$\tan 2\theta = -\frac{(H/H_{sf})^2 \sin 2}{1-(H/H_{sf})^2 \cos 2\Theta}, \quad H_{sf} = H_c k / 2\sqrt{2} \gg H_d.$$

## Out-of- PLANE FIELD 2

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i.  $(H/H_{sf})^2 \cos 2\Theta = 1, \mathbf{k}^2 = k^2/2.$

ii.  $\Theta = 0, \mathbf{H} \parallel \hat{z}.$

$$E = -S^2 J_0 - SH^2/2H_c + \Delta E.$$

$$\Delta E \sim (H^2 - H_{sf}^2) \sin^2 \theta.$$

$$H < H_{sf}, \theta = \pi/2, \text{ Helix.}$$

$$H > H_{sf}, \theta = 0, \text{ AF in flop state.}$$

# RESULTS

In-plane field rotates  $(\hat{c}, \mathbf{k})$  cross.

Strong field  $\hat{c} \parallel \mathbf{H}$ .

Two chiral domains.

Out-of-plane field.

$(k$  decreasing.

Strong field.

Helix  $\rightarrow$  Antiferromagnet.

# 3-D GENERALIZATION

GGGGGGGGGGGGGGGGGGGGGGGG

$\hat{c}$ :  $(\hat{c}, \mathbf{k})$  plane,  $[\hat{c} \times \mathbf{k}]$  direction.

plane field. Rotation of  $\mathbf{K} \pm \mathbf{k}$  reflections (

main transitions.

polarization:  $\Delta\sigma \sim (\hat{c} \cdot \mathbf{K})(\mathbf{K} \cdot \mathbf{P})$  at  $\mathbf{P} \parallel \mathbf{H}$

changes with  $\hat{c}$  rotation.

out-of-plane field.

decreasing, AF transition.

СПАСИБО!

## FM LATTICE

FM critical field  $H_f = SJ_0k^2/2$ .

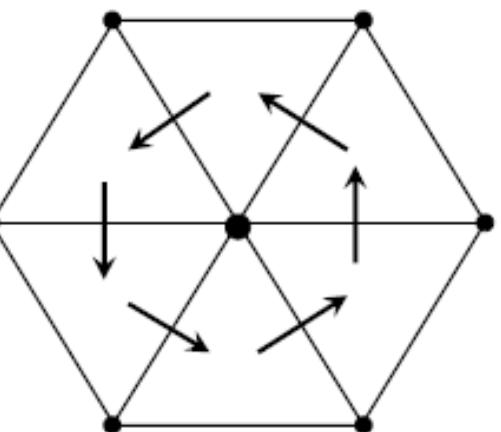
Domain rotating field

$$H_{fd} = SH_fk \sim k^3 \ll H_d \sim k^2.$$

Anisotropy: Dipolar interaction:

$$A \rightarrow A + (g\mu_B)^2 A_1/a^3, \quad A_1 \simeq 26.$$

# TRIANGULAR LATTICE



Rounds  $\mathbf{h}_o = \hat{r}$   
 $\mathbf{b}_{\pm} = \hat{x} \cos \psi \pm \hat{y} \sin \psi, \psi = \pi/3$

$$E = -2S^2 \sum_{j=0,\pm} \{ J \cos(\mathbf{b}_j \cdot \mathbf{k}) -$$

$$d[(\hat{c} \cdot [\hat{z} \times \mathbf{b}_j]) \sin(\mathbf{b}_j \cdot \mathbf{k})] \} - S(\mathbf{H} \cdot \hat{c})/2H_c,$$

# K-EXPANSION 1

$k^2 : \mathbf{k} \perp \hat{c}$  Cycloid.

$$E = -S^2 J_0 \left(1 + \frac{k^2}{4}\right) - \frac{S(\mathbf{H} \cdot \hat{c})^2}{2H_c}.$$

$k = d/J, J_0 = 6J,$

$H_a = 4SJ_0, H_f = SJ_0k^2/2.$

$$\Delta E_4 = (S^2 J_0 k^4 / 16) \sum_{n=0\pm} \sin^4(\phi + n\pi/3) = \frac{9S^2 J_0 k^4}{132},$$

# SIX ORDER ANISOTROPY

$$\Delta E_6 = -\frac{S^z J_0 k^{\nu}}{144} \sum_{n=0,\pm} \sin^6(\phi + n\pi/3) \sim \cos 6\phi,$$

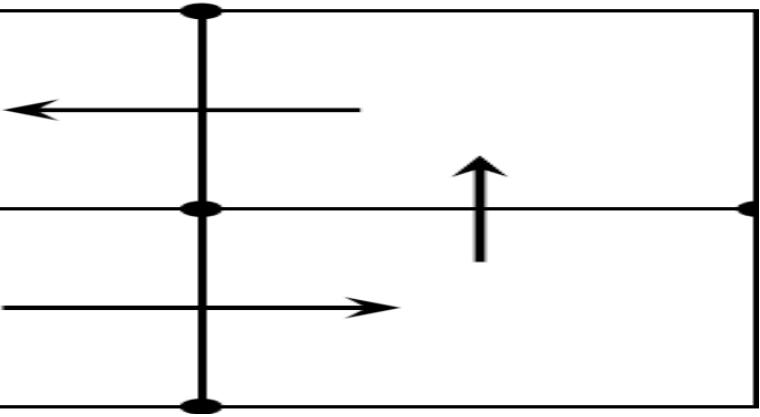
Three domain pairs.

Easy  $\hat{c}$  axes:  $\pm\pi/6, \pi/2$ .

Domain rotating field

$$H_d = \frac{\sqrt{3}H_a k^3}{96}(AF), \frac{\sqrt{3/2}H_f k^4}{48}(FM).$$

# RECTANGULAR LATTICE



$$k_x = -\frac{d_x \hat{c}_y}{J_x}, k_y = \frac{d_y \hat{c}_x}{J_y}.$$
$$E = -2S^2 \left( \frac{d_x^2}{J_x} \sin^2 \phi + \frac{d_y^2}{J_y} \cos^2 \phi \right).$$

tes:  $\phi = 0, \phi = \pi/2$

$$\frac{d_y^2}{J_y}, \hat{c}||\hat{y}, \mathbf{k}||\hat{x}..$$

$E = S^2 \left( \frac{d_x^2}{J_x} - \frac{d_y^2}{J_y} \right)$  rotates  $(\hat{c}, \mathbf{k})$

$(\hat{c}_y) \rightarrow (\hat{c}_x) ..$

# LAYER POLARIZATION

Layer is at  $z_0$  above the substrate.

Elastic energy  $U(z) = -\frac{1}{2}K(z - z_0)^2$ .

Helix energy  $E = -S^2 d^2(z)/4J_0$ .

Layer shihifting  $\delta z = -S^2 dd'/2KJ_0$ .

Polarization  $\mathbf{P} \sim \delta z \hat{z}$ .

$\delta z = 0$  if  $H_\perp > H_{sf} \sim H_c k$ .

# CONCLUSIN

Magnetic field affects strongly

the DM helical structure:

It may rotate the vector  $\mathbf{k}$ ,

Chane its value,

Supress the helix.

СПАСИБО!