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Observation of 4π-periodicity of the spinor using neutron resonance interferometry

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Abstract. – A polarised neutron beam is passed through a gradient resonance flipper. By the amplitude of their RF field, such flippers can be set at flip probability $\rho = 0$, 1, or $\frac{1}{2}$. At $\frac{1}{2}$, the neutron wave splits into a flipped and a non-flipped part with different precession. We measure the polarisation after a spin-echo (SE) setup with each precession arm made up of 2 such flippers. Offset from SE is made by varying the static fields in one flipper while the other flippers stay unchanged. This shows up as a periodic behaviour of the polarisation. With incoming polarisation parallel to the static field in the flippers —set at $\rho = \frac{1}{2}$— this period is twice the period measured for both $\rho = 0$ and 1 and with polarisation perpendicular to the static field. In the latter experiments both components of the spinor are affected, whereas in the former experiment we create spin states in which only one spinor component in one state is affected. Hence, this experiment demonstrates explicitly the 4π-periodicity of the spinor.

Introduction. – Larmor precession of the polarisation of a neutron beam in a magnetic field can be described by means of specific changes in the spinor, *i.e.* a normalised vector in 2-dimensional space with complex components:

$$|\psi\rangle = \begin{pmatrix} a\exp[i\phi_1] \\ b\exp[i\phi_2] \end{pmatrix} = a\exp[i\phi_1]\begin{pmatrix}1\\0\end{pmatrix} + b\exp[i\phi_2]\begin{pmatrix}0\\1\end{pmatrix}, \tag{1}$$

where the phases ϕ_1 and ϕ_2 characterise the spinor in its initial form. (Their value is irrelevant, since our interest concerns the change in these phases.) From this spinor the components S_i ($i = x, y, z$) of the average spin (=polarisation) are calculated according to $S_i = \langle\psi|\sigma_i|\psi\rangle$, where σ_i is the component i of the Pauli matrix vector $\vec{\sigma}$. The most general unitary operator to apply to the spinor in order to describe Larmor precession over an angle α around a field in the direction of the unit vector $\vec{n}$, takes the form $\hat{R} = \exp[-i\vec{\sigma}\cdot\vec{n}\ (\alpha/2)]$. By expanding the exponential this operator can be shown to be equal to

$$\hat{R} = \cos(\alpha/2)\hat{I} - i\vec{\sigma}\cdot\vec{n}\sin(\alpha/2), \tag{2}$$

where $\hat{I}$ is the (2×2) identity matrix. If we choose the coordinate system such that the field is parallel to z, the dot product $\vec{\sigma}\cdot\vec{n}$ reduces to $\sigma_z = \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right)$, so applying this operator to the spinor

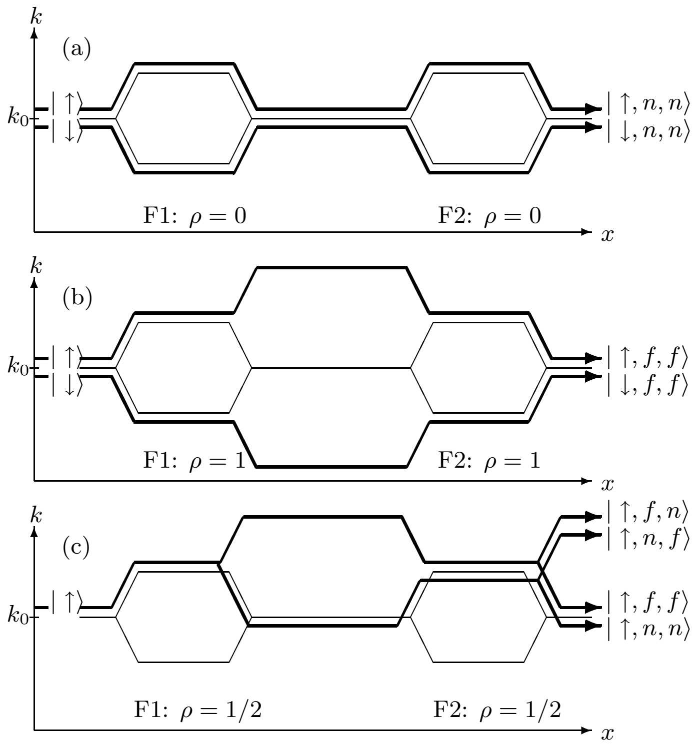

Fig. 1 – (k, x) diagram for the first arm of a spin-echo setup consisting of 2 neutron resonance spin flippers. The thin lines schematically show the splitting of the wave vector $k(x)$ for the initial states $|\uparrow\rangle$ and $|\downarrow\rangle$ in the fields of the flippers F1 and F2 in case no flip happens. The thick lines mark "pathways" in the (k, x)-space, as discussed in the text. (The actual flippers contain gradient fields giving a slight modification of these lines which is irrelevant here.)

means that we *add* $\alpha/2$ to the phase of the component along $\binom{1}{0}$ and simultaneously *subtract* $\alpha/2$ from the phase of the component along $\binom{0}{1}$. When we calculate the components of the average spin $\vec{S}$, we find that this vector has rotated by α around the z-axis. Taking $\alpha/2 = 2\pi$, this means that a full period of 2π for the components of the spinor gives a rotation of the "observable" polarisation vector $\vec{S}$ over 4π. Hence, the recovery of the initial spinor is obtained only after 4π rotation of the polarisation. This is called the 4π periodicity of the spinor.

The common way to add/subtract a certain phase in the components of the spinor is to subject the neutron beam over some path length to a magnetic field B. The neutron wave with initial wave number k_0, once in the field, splits into plane waves corresponding to the spin-up $\binom{1}{0}$ (or $|\uparrow\rangle$) and spin-down $\binom{0}{1}$ (or $|\downarrow\rangle$) states with wave numbers $k^+ = k_0 + \frac{2\mu_n B}{\hbar v}$ and $k^- = k_0 - \frac{2\mu_n B}{\hbar v}$ (μ_n = magnetic moment, v = velocity of the neutrons). Their phases increase at different rates. At the end of the field k^+ and k^- return to k_0, so from this point on the phases grow again at equal rates. The thin lines in fig. 1a illustrate this for a succession of 2 DC magnets (x is the travelling direction of the waves). The phase acquired by the terms for $\binom{1}{0}$ and $\binom{0}{1}$ in the spinor equals $\int(k^+(x) - k_0(x))\mathrm{d}x$ and $\int(k_0(x) - k^-(x))\mathrm{d}x$, respectively. The polarisation precessed over an angle equal to the sum of these integrals. It is the area between the thin lines for k^+ and k^- marked by thick lines. The $|\ldots\rangle$ symbols indicate that no flip (n) happened in the magnetic fields labelled F1 and F2. This mode of precession is called "DC mode".

When a spin flipping device sits in the first field and spin flip happens, the wave numbers k^+ and k^- jump to $k^{++} = k^+ + \frac{2\mu_n B}{\hbar v}$ and $k^{--} = k^- - \frac{2\mu_n B}{\hbar v}$ upon leaving the field. This means that the phase difference between the waves increases as a function of x twice as fast as in the DC mode. This is called "zero-field precession" by Gähler *et al.* [1–3]. We refer to it as "RF mode". To halt this precession, one needs spin flip by a flipping device in the second magnetic field, which returns k^{++} and k^{--} to k_0. In the (k, x)-space this mode of precession is represented by the thick lines in fig. 1b. The $| \ldots \rangle$ symbols indicate that flip (f) happened. We produced this mode of precession for the full white spectrum, by passing the polarisation in adiabatic way through 2 gradient NR flippers [4,5]. The above descriptions of DC and RF precession lead to the idea of a *pathway* of a neutron wave in the (k, x)-space.

One could imagine to increase the phase of only one component of the spinor by α and leave the other component unchanged. This would produce a precession of the polarisation vector $\vec{S}$ about the field direction (z) over α, in other words, the observed period of the polarisation would be *equal* to the period of the spinor. This was done by several authors in neutron interferometers. They modified the precession phase along one of 2 spatially separated paths by a magnetic field along that path [6–9] and thus demonstrated the 4π periodicity of the spinor in response to the magnetic field.

The aim of this paper is to demonstrate this in the (k, x)-space. For an interference experiment in the (k, x)-space we consider the diagrams in fig. 1 as the first arm of a neutron spin echo (SE) interferometer. A second SE arm (which is left unchanged) compensates the phases of the waves in the first arm, which do change when we vary parameters acting on the phase of waves travelling along different paths in the (k, x)-space.

In the experiments of fig. 1a and b, the polarisation of the beam at entrance was perpendicular to the field direction, which means that we feed the initial states $| \uparrow\rangle$ and $| \downarrow\rangle$ equally. Now, let us operate the flippers at probability $\frac{1}{2}$ (called "DC/RF mode") and feed the interference experiment with only state $| \uparrow\rangle$ (by aligning the incoming polarisation not perpendicular, but parallel to the field). The initial single state $| \uparrow\rangle$ will double after each flipper. This means that we "realise" the pathways marked as thick lines in fig. 1c. This is the technique of "separated coils" introduced by Ramsey [10]. Of the 4 pathways after flipper 2, only the phase difference between the pairs $[| \uparrow, f, f\rangle - | \uparrow, n, n\rangle]$ and $[| \downarrow, f, n\rangle - | \downarrow, n, f\rangle]$ can be observed. (We observed these interferences in earlier experiments [4,11].) The phase difference between the other combinations oscillates in time and will average out in the static experiments discussed below. So, neutrons, as far in the states involved in these interferences add as background in the observed intensities. Below, we explain that gradient NR flippers provide parameters which "work" on only one state of the pairs mentioned above.

Gradient NR flipper. – The flippers in this experiment consist of a *static* field along the neutron path $x = [0, l]$ (l = length of the flipper), written as $B(x) = B_0 + A_{\rm gr}\cos(\pi x/l)$, where the cosine term is a *gradient* field added to the homogeneous field B_0). Such a gradient field is absent in a mere resonant flipper [12]. Superposed on this is a longitudinal field *oscillating* at frequency $\nu_{\rm rf}$ such that the resonant point ($2\pi\nu_{\rm rf} = \frac{\mu_n B_0}{\hbar} \equiv \gamma B_0$) is near the center of the flipper. The field $B_{\rm rf}$ must vary along the range $x = [0, l]$ from 0 at $x = 0$ to a maximum halfway and back to 0 at $x = l$: $B_{\rm rf}(x) = A_{\rm rf}\sin(\pi x/l)\exp[i\ 2\pi\nu_{\rm rf}t]$.

When the resonance condition is fulfilled, the neutrons, as seen in the frame $(\tilde{x}, \tilde{y}, z)$ rotating at the frequency $\omega_0 = 2\pi\nu_{\rm rf}$ about the z-axis, are affected by the sum of two fields: the static gradient field pointing along the z-axis —reduced by the value B_0—, and the oscillating field $B_{\rm rf}$ which in this system also appears static. For a neutron flying with velocity v, the effective field $B_{\rm eff}$ rotates in the $\tilde{x}z$ plane with frequency $\Omega = \pi/\tau$, where $\tau = l/v$ is the time which the neutron needs to pass this interval. During this time the spin rotates about $B_{\rm eff}$ at a frequency $\omega_L = \gamma A$, where A is the magnitude of the effective field. If A is large enough, *i.e.* the adiabatic condition $\omega_L \gg \Omega$ is satisfied (or the adiabaticity parameter $k \equiv \gamma Al/(\pi v) \gg 1$), the neutron spin follows the effective field. Back in the laboratory system, this means that spin is reversed.

The *spin flip probability* ρ for such a configuration is [4, 12]

$$\rho = 1 - \sin^2 \phi / \left(k^2 + 1\right), \tag{3}$$

where ϕ is the phase of the spin in the magnetic field of the rotating frame. ρ may be readily changed between 1 and 0 by changing the amplitude A_{rf} of the oscillating field from some maximum to 0, *i.e.* by changing the adiabaticity parameter k from $\gg 1$ to 0.

For the *precession phase* we must distinguish between f and n. The *non-flipped* part of the spinor neither gains nor loses energy. This means that it did not interact with B_{rf}. Its phase is

$$\Delta\phi_n = \frac{\gamma}{v}\int_0^l B_0(x)\mathrm{d}x = \frac{\gamma}{v}\int_0^l B_0 + A\cos(\pi x/l)\mathrm{d}x = \frac{\gamma}{v}B_0 l. \tag{4}$$

The phase for the *flipped* part of the neutron wave in our magnetic-field configuration is $\Delta\phi_f = \omega_0\tau + (\pm)\phi = \omega_0\tau + (\pm)(\pi\sqrt{k^2+1})$, as was shown in [2] for the case $A_{\mathrm{gr}} \approx A_{\mathrm{rf}} \approx A$ and the adiabatic condition fulfilled ($k \gg 1$). The term $\omega_0\tau$ is the contribution of the rotating frame, as in a conventional flipper [1, 3–5]. The second term is the precession phase itself in the rotating frame. Its sign depends on the sign of the gradient field with respect to the spin. We can rewrite the phase $\Delta\phi_f$ as

$$\Delta\phi_f \approx \omega_0\tau + \frac{\gamma}{v}\int_0^l |B_{\mathrm{eff}}(x)|\mathrm{d}x = \omega_0\tau + \frac{\gamma}{v}\int_0^l \sqrt{B^2_{x,\mathrm{eff}}(x) + B^2_{z,\mathrm{eff}}(x)}\mathrm{d}x, \tag{5}$$

where $B_{x,\mathrm{eff}}(x) = A\sin(\pi x/l)$ and $B_{z,\mathrm{eff}}(x) = B_0 - \omega_0/\gamma + A\cos(\pi x/l)$. In principle the field B_0 is chosen such that the terms B_0 and ω_0/γ cancel, but in practical reality as long as $|B_0 - \omega_0/\gamma| < A/2$, the flipper will work, so eq. (5) remains valid and a second-order effect on $\Delta\phi_f$ due to a variation of B_0 will be present. We neglect it for the purpose of this work.

Combining these equations for the case of incomplete flip, we conclude:

1) the constant permanent field B_0 determines the phase of the non-flipped part of the wave and has a second-order effect on the phase of the flipped part;

2) the amplitudes A_{gr} and A_{rf} as set by the experimentalist determine the phase of the flipped part of the neutron wave, but not of the non-flipped part.

Layout of the NRSE experiment. – The setup is shown schematically in fig. 2. A polychromatic polarized neutron beam enters rotator R1 where the polarization can be rotated towards the y-axis ($\perp$ field direction in SE arms) or kept parallel to the initial direction z (= field direction in SE arms). Behind the SE setup sits a mirrored rotator R2. The combined rotators allow to apply and analyze the polarisation perpendicular (denoted P_{yy}) or parallel to the field direction (denoted P_{zz}). In addition, rotator R2 allowed for measuring in 2 anti-parallel modes, which in any setting enabled us to calculate the beam polarisation. Spin-echo arm 1 is a set of two gradient NR spin flippers F1 and F2 at center-to-center distance 0.9 m. Details of their construction are given elsewhere [13]. To "smooth" the field gradients between the flippers and for guide fields, iron plates are mounted below and above the beam axis. Spin-echo arm 2 is identical with 1, but with opposite static field. The current sheet CSh produces a stepwise field transition between the SE arms.

In each flipper we could independently vary the parameters: magnetic field B_0 (0–1000 G), amplitude A_{gr} of the gradient field (0–40 G), and amplitude A_{rf} of the RF field (0–20 G). Data

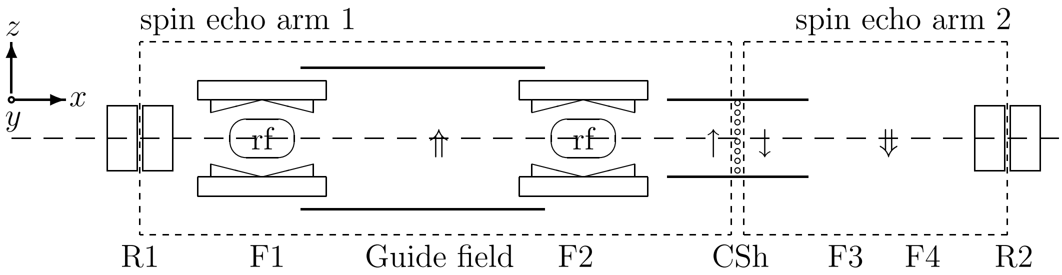

Fig. 2 – Schematic side view of the spin-echo (SE) interferometer installed between a polariser and analyser (not shown). The ovals "rf" in the flippers F1 and F2 in the first SE arm represent the longitudinal RF coils, the triangles represent the gradient coils. The second arm is schematized.

were collected in a detector bank placed in the reflected beam of a monochromator crystal behind the analyzer. The wavelength in various detectors ranged from $\lambda = 0.19 \ldots 0.23\,\mathrm{nm}$ with a spread $\simeq 0.02\,\mathrm{nm}$.

Setting flipping probability. – To find how to set the flipping probability ρ, we first measured ρ in mode P_{zz} for each flipper as a function of A_{gr} and A_{rf}, in the way published in [13]. As an example, fig. 3 shows results for F1. One sees that $\rho \simeq 1$ (exceeds 0.85) for $A_{\mathrm{rf}} \approx 12\,\mathrm{G}$ (upper edge of the map) and that $\rho \simeq 0.5$ for $A_{\mathrm{rf}} = 4\,\mathrm{G}$, both irrespective of the value of the gradient amplitude.

Interference experiments. – Prior to each experiment the parameters of all flippers were set and the SE interferometer was balanced by means of a "phase coil" in SE-arm 1. In the experiments we varied the parameters of flipper F1, the other flippers being unchanged.

First, following the scheme of fig. 1a, with all flippers off ($\rho = 0$, DC mode), we change the gradient amplitude in flipper F1. Equation (4) predicts that this will affect the phases of neither $|\uparrow, n, n\rangle$ nor $|\downarrow, n, n\rangle$, so the polarisation (when the rotators are set for measuring P_{yy}) will not vary. This is shown in fig. 4a for 2 detectors.

Next, all flippers are set to flip probability $\rho = 1$ (RF precession mode), fig. 1b. According to eq. (5) the phases of both the states $|\uparrow, f, f\rangle$ and $|\downarrow, f, f\rangle$ will change, hence the polarisation varies periodically as a function of the gradient amplitude of F1, as shown in fig. 4b.

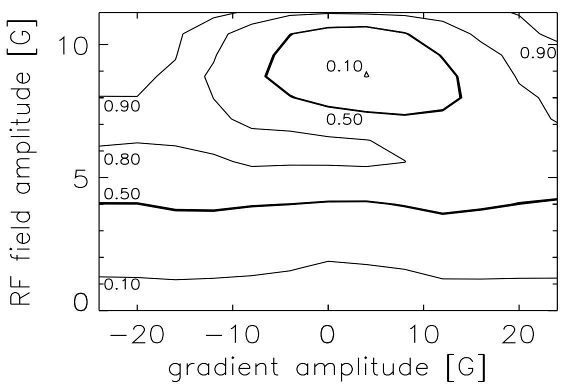

Fig. 3 – Flipper F1: map of the flipping probability ρ at $B_0 = 414\,\mathrm{G}$, $\lambda = 0.193\,\mathrm{nm}$, showing the locus of points where $\rho = \frac{1}{2}$ (thick lines).

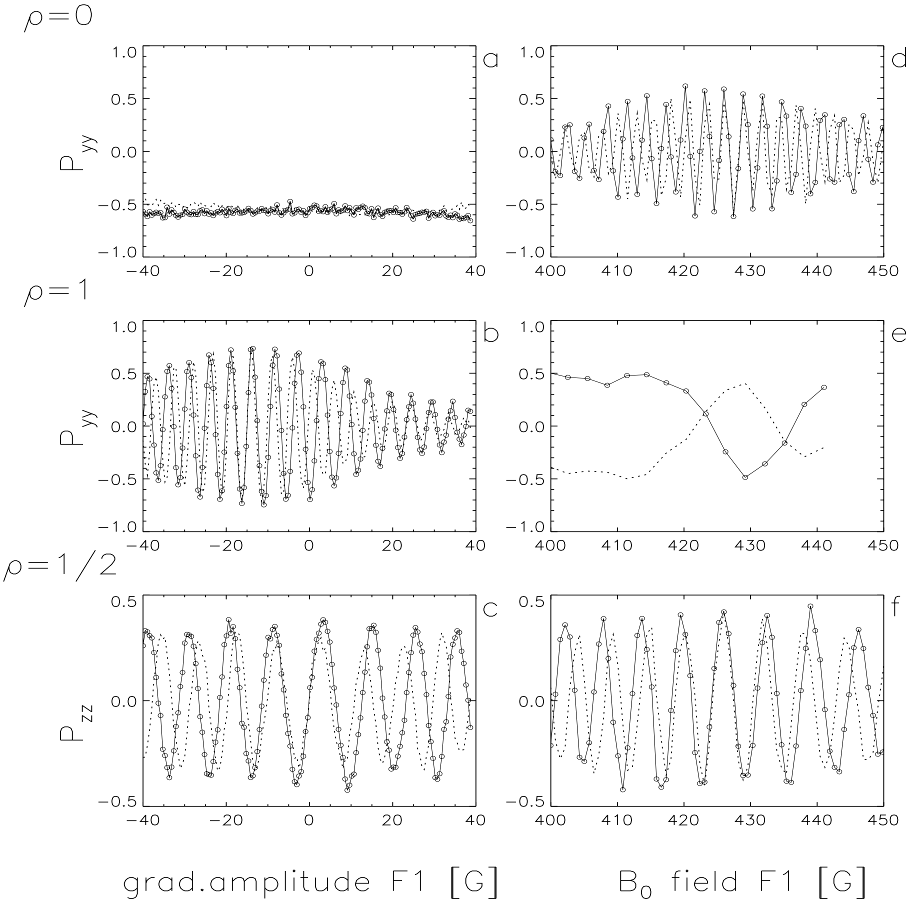

Fig. 4 – Polarisations measured in interference experiments for the pathways through the (k, x)-space of fig. 1, while varying the parameters gradient A_{gr} (left) and constant field B_0 (right) of flipper F1. Full lines: detector observing $\lambda = 0.193\,\mathrm{nm}$; dotted lines: idem $\lambda = 0.217\,\mathrm{nm}$.

Now we do the same, with rotators R1 and R2 set for measuring P_{zz}, with the flippers at $\rho = \frac{1}{2}$ (DC/RF mode, fig. 1c). Varying the gradient amplitude of flipper F1 will affect $|\uparrow, f, f\rangle$, but not $|\uparrow, n, n\rangle$. Therefore, the phase difference between these 2 states changes at half the rate of the previous experiment, so the polarisation will vary with the *double* period. This is confirmed by the result in fig. 4c.

A similar set of experiments can be done by varying the permanent field B_0 of flipper F1. In the DC precession mode (fig. 1a) the phases of $|\uparrow, n, n\rangle$ and $|\uparrow, n, n\rangle$ change and we see a periodic variation of the polarisation P_{yy} (fig. 4d). No phase variation is seen (in first order) in RF precession mode (fig. 1b), which is shown in fig. 4e. Again, for the polarisation P_{zz}, the observed period in B_0 doubles, when the flippers are operated at $\rho = \frac{1}{2}$ (DC/RF mode), because $|\uparrow, n, f\rangle$ is affected but not $|\uparrow, f, n\rangle$. This is shown by fig. 4f.

In both sets of experiments the *absolute* period of the signals can be accounted for (to a precision of 20%) on the basis of eq. (5) and the known profiles of the static B_0 field and of the gradient field [13]. In the interpretation this imprecision plays no role, since we observed 2 distinct periods in both sets of experiments with a *ratio* equal to 2 within 2%.

Interpretation. – The spinor, represented as the vector $\binom{\alpha \exp[i\phi_1]}{\beta \exp[i\phi_2]}$, may be affected by three different tools, which are driving parameters of the gradient NR spin flipper: $B_{\rm rf}$, and $A_{\rm gr}$, and B_0. We selected the parameter $B_{\rm rf}$ to set the spin flip probability ρ equal to 0, 1, or $\frac{1}{2}$, in order to observe the spinor behavior in the modes DC (fig. 1a), RF (fig. 1b), or DC/RF (fig. 1c), respectively.

The parameter $A_{\rm gr}$, in the DC mode, lets the spinor unchanged; in the RF mode it is changed into $\binom{\alpha \exp[i\phi_1 + i\chi(A_{\rm gr})]}{\beta \exp[i\phi_2 - i\chi(A_{\rm gr})]}$ and in the DC/RF mode into $\binom{\alpha \exp[i\phi_1 + i\chi(A_{\rm gr})]}{\beta \exp[i\phi_2]}$. Here $\chi(A_{\rm gr})$ is the phase shift in the flipped part of the neutron wave. In terms of the observables one gets: $P \sim \cos(\phi_1 - \phi_2)$ for the DC mode (fig. 4a); $P \sim \cos[(\phi_1 - \phi_2) + 2\chi(A_{\rm gr})]$ for the RF mode (fig. 4b); and $P \sim \cos[(\phi_1 - \phi_2) + \chi(A_{\rm gr})]$ for the DC/RF mode (fig. 4c).

The same consideration applies when the parameter B_0 varies. In observables one gets: $P \sim \cos[(\phi_1 - \phi_2) + 2\chi(B_0)]$ for the DC mode (fig. 4d); $P \sim \cos(\phi_1 - \phi_2)$ for the RF mode (fig. 4e); and $P \sim \cos[(\phi_1 - \phi_2) + \chi(B_0)]$ for the DC/RF mode (fig. 4f) with $\chi(B_0)$ as a phase shift produced by the permanent field B_0 in the unflipped part of the neutron wave.

This consideration demonstrates that in the DC/RF mode the observable P shows the "true" periodicity of the spinor, while in the RF and DC modes the periodicity of observable is twice less than that of the spinor. Furthermore, it has been shown in many experiments that the observable P changes periodically under a magnetic field with 2π-periodicity. Therefore, we can conclude that in the DC/RF mode one observes the 4π-periodicity of the spinor.

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