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Physica B 335 (2003) 243–246
PHYSICA B
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Neutron multiwave interference experiments with many resonance coils

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Abstract

Neutron multiwave interference phenomena based on Ramsey's resonance method of "separated oscillating fields" are studied. The neutron passes through N successive resonant coils (with DC field B_0) which flip the neutron spin with a probability ρ (written as $\rho = \sin^2 \xi$) smaller than 1. These coils are separated by path lengths l over which a homogeneous field B_1 is present. Because the spin-flip probability ρ is smaller than 1, the number of states is doubled after each flipper, so as to produce 2^N neutron waves at the end of the set-up. The phase difference between any pair of waves is a multiple of a "phase quantum" equaled to the line integral over length l of the field difference $B_1 - B_0$. Highly regular patterns of the quantum mechanical probability R in (B_1, ξ)-space appear due to pair interference between individual waves.

PACS: 03.75.D; 42.87.B

Keywords: Neutron multiwave interference

1. Introduction

Multipath interference in optics and multimode interference in dynamical systems have recently emerged as an extremely active field of research. The regular structure of the probability density for propagation of the Gaussian wave packet in time and space, the so-called quantum carpet, becomes an object of study [1]. Therefore, it is of great interest to prepare a wave packet in a controlled way and to measure its interference while propagating along various paths or in various modes. In this paper, we use Ramsey's resonance method of the "separated oscillating fields" [2] to study neutron multipath wave interference. Recently, we carried out experiments to demonstrate interference of two neutron waves having different prehistories in spin-momentum space [3]. It should be noticed that this phenomenon has much in common with the neutron resonant spin echo (NRSE) method [4–6].

2. Theoretical background

We study the probability density for a neutron passed through N resonant coils which flip the

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0921-4526/03/$ - see front matter
doi:10.1016/S0921-4526(03)00247-3

neutron spin with a probability ρ between 0 and 1. They are embedded in a homogeneous field B_0 and operated at the resonance frequency ($\hbar\omega_0 = 2\mu_n B_0$). They are separated one from another by path lengths l in which a homogeneous field B_1 is present (see Fig. 1a). Let us assume a plane neutron wave travelling along the x-axis through this configuration of magnetic fields. The neutron beam is polarised in a direction parallel to B_0.

As soon as the neutron enters the field B_0, the wavenumber k_0 changes to k_1. Taking energy conservation into account, one can write in the first approximation: $k_1 = k_0 + \mu_n B_0/(\hbar v)$. Here, m_n, μ_n and v are the mass, the magnetic moment and the velocity of the neutron, respectively. If spin flip occurs, the total energy of the neutrons is not conserved because of exchange of a photon of energy $\hbar\omega_{RF}$ between the neutron state and the oscillating or RF-field. During the spin flip in the RF field, neutron spin states with wavenumber k_1, will gain or lose an amount of potential energy $\Delta E = 2\mu_n B_0$. Then, upon moving from the static field B_0 to B_1, potential energy is released as a kinetic energy change. Since the spin flip is not complete but only partial, the neutron wave is split into two plane waves with wavenumber k_+ and k_- corresponding to spin state $\binom{1}{0}$ (up) and $\binom{0}{1}$ (down), respectively. Again, due to the law of conservation of energy, the total energy of each wave does not change at the transition from B_0 to B_1 and therefore k_+ and k_- become in first approximation:

$$k_\pm = k_1 \pm \frac{\mu_n \Delta B}{\hbar v} = k_0 + \frac{\mu_n (B_0 \pm \Delta B)}{\hbar v}, \qquad (1)$$

where $\Delta B = B_1 - B_0$.

After the second coil each wave is split again into two equally populated waves, and so on. Thus after N resonance coils the initial (plane) wave is split into two groups of 2^{N-1} neutron waves with amplitudes equal to $1/(2^N)$ the amplitude of the initial wave. The waves of the first group have now energy $\hbar(\omega + \omega_{RF})$, they were flipped an odd number of times and therefore have the spin state "down". The other group of 2^{N-1} waves with amplitudes $1/(2^N)$ has energy $\hbar\omega$, as the initial wave had. They were flipped an even number of times; one of them was not flipped at all. Therefore they have the spin state "up". So, finally one group of waves is located at the upper level of the (k, x) diagram (Fig. 1b) with its own energy and spin state, and the other group is at the lower k-level with a different energy and opposite spin state.

In fact, the neutron waves of each group differ only in phase, since each wave has its own unique path in the $(k - x)$ diagram (Fig. 1c). Each pair of waves interferes; the phase difference for an arbitrary pair is $\phi = m\Delta\phi$, where $m = 0, 1, \cdots, N$ and

$$\Delta\phi = \int_0^l (k_+(x') - k_-(x'))\,\mathrm{d}x' \qquad (2)$$

is the integral over one length l with field B_1. Thus, $\Delta\phi$ is a quantum of phase. The population number A_i ($i = 1 \ldots 2^N$) of an arbitrary wave depends on the number of flip m and non-flip $(N - m)$ events

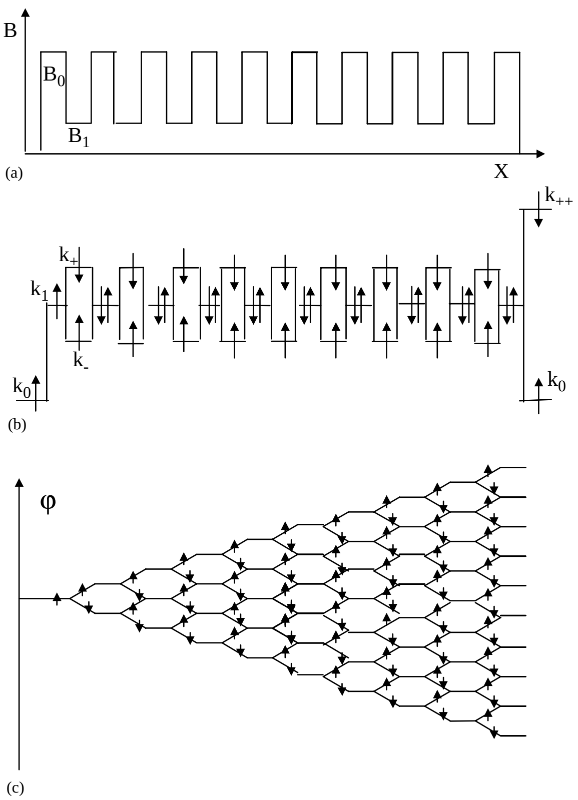

Fig. 1. (a) Sketch of the system with many RF spin flippers with a guide field between them. (b) (k, x) diagram of the wave vectors as a function of position along the beam. (c) Diagram of the phase of the wave function along the beam.

by which it was created, so that $A_i = \rho^{m/2}(1-\rho)^{(N-m)/2}$, $m = 0, \ldots, N$. All pairs combined give rise to a highly regular pattern in quantum mechanical probability.

3. Numerical experiment

An analytical expression for the redistribution of the quantum mechanical (QM) probability over the two energy levels after passage through a large number N of resonant coils is difficult to obtain. It is especially sophisticated for an arbitrary value of the spin-flip probability ρ of the resonant coils. Nevertheless, in order to testify to the above theoretical consideration, a computer simulation was performed. The computation technique is based on a successive multiplication of (2×2) matrices (denoted $\hat{C}$ [2,3]) which describe the action of the resonant coil and of matrices (denoted $\hat{H}$) which describe the action of the pseudomagnetic field ΔB in the space between the coils [4–6]). From the resultant neutron wave function the QM probability R was obtained. It depends on two system parameters that one can vary.

The first one is the spin-flip probability ρ, written as $\rho = \sin^2 \xi$. Here, $\xi = \gamma_n B_{RF} l_{RF}/2v$ where B_{RF} and l_{RF} are the field amplitude and the length of the RF coil and v is the neutron velocity. By varying B_{RF} one can vary ξ from 0 to $n\pi$ where n is an integer number. Another important parameter is the phase quantum $\Delta\phi$, that is proportional to ΔB.

The picture of the QM probability is ruled by the number N of resonance coils. For $N = 2$ and $\rho = \frac{1}{2}$ the QM probability has a cos dependence on $\Delta\phi$, in agreement with Ramsey's calculations [2]. As N is increased, a sophisticated pattern appears with a periodicity of π along the $\Delta\phi$ axis. Additional peaks appear between two main maxima. Their number n_{sec} depends on the number N according to the simple relation: $n_{sec} = (N-2)/2$.

For $\rho \neq \frac{1}{2}$ the occupation numbers of the different interfering waves become different. Fig. 2 shows the dependence of the QM probability R on the phase $\Delta\phi$ for $N = 10$; the argument ξ in the spin-flip probability $\rho = \sin^2 \xi$

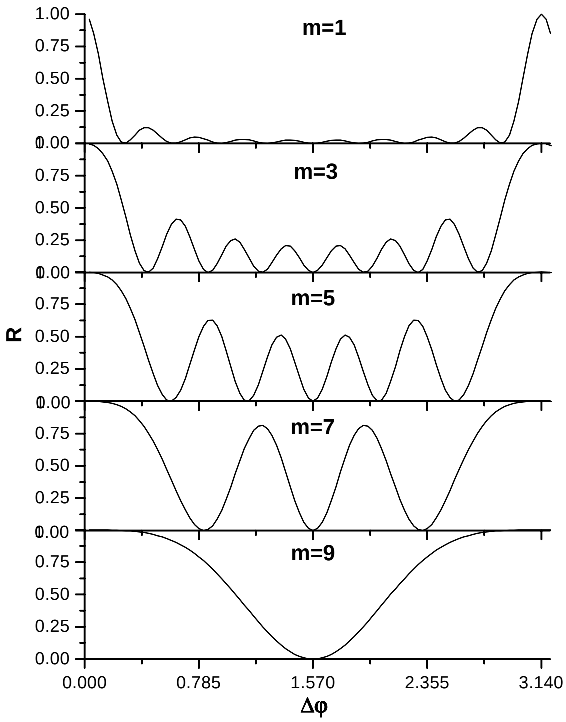

Fig. 2. Dependence of the QM probability R for a system of $N = 10$ coils on the phase quantum $\Delta\phi$ (Eq. (2)) and on the spin-flip probability ρ written as $\sin^2 \xi$ where $\xi = m\pi/(2N)$ with $m = 1, 3, 5, 7, 9$.

was taken equal to $m\pi/(2N)$ with $m = 1, 3, 5, 7, 9$. For these values of ξ, the QM probability has maxima at $\Delta\phi = n\pi$, i.e. it is periodically self-reconstructed. The number of secondary maxima between the main maxima decreases, but their height increases as ξ grows by increasing m.

Fig. 3 shows the QM probability R as a function of the phase ξ for $N = 10$ and for phase quantum value $\Delta\phi = k\pi/N$ with $k = 0, 1, 2, 3, 4$. In the top picture ($\Delta\phi = 0$) 10 maxima appear in accordance with the number N of resonant coils. The number of maxima decreases as $\Delta\phi$ increases from 0 to $\pi/2$. Two peaks per step $\Delta\phi = \pi/N$ vanish. For $\Delta\phi = \pi/2$ the QM probability R becomes equal to 0. A further increase of the phase quantum $\Delta\phi$ results in a full reconstruction of the pattern at $\Delta\phi = \pi$.

As a conclusion, we see that the system of N resonant coils produces a large number of neutron

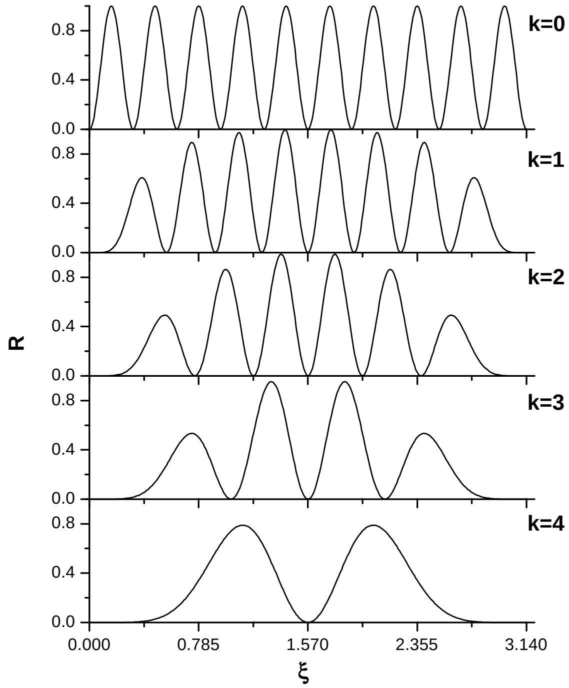

Fig. 3. Dependence of the QM probability R for a system of $N = 10$ coils on the phase ξ and on the phase quantum $\Delta\phi = k\pi/N$ with $k = 0, 1, 2, 3, 4$.

waves. These waves interfere and each pair of them contributes to highly regular patterns in quantum mechanical probability. The analytical expression for QM probability is a sum of cos functions. The problem of multiwave interference may be of great interest both from theoretical and experimental points of view.

Acknowledgements

This work is part of the research programme of the “Stichting voor Fundamenteel Onderzoek der Materie (FOM)”, which is financially supported by the “Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)”. The authors thank RFFR (Project 00-15-96814) and the Russian State Programme “Neutron Research of the Condensed State”.

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