Законы сохранения известные и неизвестные в классической и релятивистской физике

С. Н. Манида, СПбГУ

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The right relativity for our universe

- Galilean relativity
- Special relativity
- Double (deformed) special relativity
- de Sitter (special) relativity
- Anti-de Sitter (special) relativity
- ....

Nonrelativistic free motion symmetry: $S = \int \sum \frac{m\mathbf{v}^2}{2} dt$

1. 3 space translations: $\mathbf{r}' = \mathbf{r} - \boldsymbol{\rho}$

$$\sum m\mathbf{v} = \mathbf{P} = \sum \mathbf{p}$$

2. time translation: $t' = t + \tau$

$$E = \sum \frac{m\mathbf{v}^2}{2} = \sum \frac{\mathbf{p}^2}{2m}$$

3. rotations: $\mathbf{r}' = \mathbf{r} + 2\mathbf{n}(\mathbf{n}\cdot\mathbf{r})\cos^2\frac{\alpha}{2} + (\mathbf{n}\times\mathbf{r})\sin\alpha$

$$\mathbf{J} = \sum m(\mathbf{r}\times\mathbf{v}) = \sum (\mathbf{r}\times\mathbf{p})$$

Nonrelativistic free motion symmetry: $$S = \int \sum \frac{m\mathbf{v}^2}{2} dt$$

4. Galilean boosts: $\mathbf{r}' = \mathbf{r} - \mathbf{u}t$

$$\sum m(\mathbf{r} - \mathbf{v}t) = \sum \mathbf{k} = \mathbf{K}(0)$$

$$\sum m\mathbf{r} \equiv \mathbf{K}(t) = \mathbf{K}(0) - \mathbf{P}t$$

Nonrelativistic free motion symmetry: $S = \int \sum \frac{m\mathbf{v}^2}{2} dt$

5. dilatation: $t' = \alpha^2 t$ $\mathbf{r}' = \alpha \mathbf{r}$

$$\sum m\mathbf{v}(\mathbf{r} - \mathbf{v}t) = \sum \frac{1}{m}(\mathbf{p} \cdot \mathbf{k}) = F(0)$$

$$F(t) = \sum m\mathbf{v}\mathbf{r} = F(0) + 2tE$$

$$\mathbf{r}(t) = \mathbf{r}(0) + \mathbf{u}t$$

$$\frac{\mathbf{r}(t)}{t} = \mathbf{r}_0 \frac{1}{t} + \mathbf{u} \qquad \frac{1}{t} = \frac{\tilde{t}}{T^2}; \qquad \frac{\mathbf{r}(t)}{t} = \frac{\tilde{\mathbf{r}}(\tilde{t})}{T}$$

$$\tilde{\mathbf{r}}(\tilde{t}) = \frac{\mathbf{r}_0}{T}\tilde{t} + \mathbf{u}T; \qquad \frac{\mathbf{r}_0}{T} \equiv \tilde{\mathbf{u}}, \qquad \mathbf{u}T \equiv \tilde{\mathbf{r}}(0)$$

$$\tilde{\mathbf{r}}(\tilde{t}) = \tilde{\mathbf{r}}(0) + \tilde{\mathbf{u}}\tilde{t}$$

Nonrelativistic free motion symmetry: $S = \int \sum \frac{m\mathbf{v}^2}{2} dt$

6. inverse time translation: $\frac{1}{t'} = \frac{1}{t} + \alpha; \qquad \frac{\mathbf{r}'}{t'} = \frac{\mathbf{r}}{t}$

$$t' = \frac{t}{1+\alpha t}; \qquad \mathbf{r}' = \frac{\mathbf{r}}{1+\alpha t}$$

$$\sum m \frac{(\mathbf{r} - \mathbf{v}t)^2}{2} = \sum \frac{\mathbf{k}^2}{2m} = I(0)$$

$$I(0)=\sum m\frac{(\mathbf{r}-\mathbf{v}t)^2}{2}$$

$$I(0)=\sum m\frac{\mathbf{r}^2}{2}-t\sum m(\mathbf{rv})+t^2\sum m\frac{\mathbf{v}^2}{2}$$

$$I(t)\equiv\sum m\frac{\mathbf{r}^2}{2}=I(0)+tF(t)-t^2E$$

$$I(t)=I(0)+tF(0)+t^2E$$

L. O'Raifeartaig and V. V. Sreedhar, **The maximal kinematical invariance group of fluid dynamics and explosion–implosion duality**; Annals Phys. 293 (2001) 215-227
***O. Jahn, V. V. Sreedhar* The Maximal Invariance Group of Newtons's Equations for a Free Point Particle,** Am.J.Phys. 69 (2001) 1039-1043

Классическая механика – это предельный случай релятивистской механики.

Почему не все законы сохранения классической механики следуют из релятивистской?

Инерциальность пространства

$$\frac{d^2\mathbf{r}}{dt^2} = 0$$

$$ct(s) \equiv x^0(s); \qquad r_i(s) \equiv x^i(s)$$

$$\frac{d^2 x^\mu}{ds^2} = 0$$

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\tau} \frac{dx^\alpha}{ds} \frac{dx^\tau}{ds} = 0$$

$$\frac{d}{ds}\left(\frac{dx^\mu}{ds} \middle/ \frac{dx^\nu}{ds} \right) = 0$$

$$\frac{d^2 x^\mu}{ds^2} \frac{dx^\nu}{ds} = \frac{d^2 x^\nu}{ds^2} \frac{dx^\mu}{ds}$$

Galilean symmetry

$$\mathbf{r}' = \mathbf{r} - \boldsymbol{\rho}, \qquad \mathbf{u}' = \mathbf{u} - \mathbf{v},$$

Relativistic symmetry as (c-)deformation of Galilean symmetry

$$\mathbf{r}' = \mathbf{r} - \boldsymbol{\rho}, \qquad \mathbf{u}_{\|}' = \frac{\mathbf{u}_{\|} - \mathbf{v}}{1 - \frac{\mathbf{u}_{\|}\mathbf{v}}{c^2}}, \qquad \mathbf{u}'_{\perp} = \frac{\mathbf{u}_{\perp}\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}{1 - \frac{\mathbf{u}_{\|}\mathbf{v}}{c^2}}$$

Second (R-)deformation of Galilean symmetry

$$\mathbf{r}'_{\|} = \frac{\mathbf{r}_{\|} - \boldsymbol{\rho}}{1 - \frac{\mathbf{r}_{\|}\boldsymbol{\rho}}{R^2}}, \qquad \mathbf{r}'_{\perp} = \frac{\mathbf{r}_{\perp}\sqrt{1 - \frac{\rho^2}{R^2}}}{1 - \frac{\mathbf{r}_{\|}\boldsymbol{\rho}}{R^2}}, \quad t' = \frac{t\sqrt{1 - \frac{\rho^2}{R^2}}}{1 - \frac{\mathbf{r}_{\|}\boldsymbol{\rho}}{R^2}}, \qquad \mathbf{u}' = \mathbf{u} - \mathbf{v},$$

1. Galilean transformations

$$t' = t; \qquad \mathbf{r}' = \mathbf{r} - \mathbf{u}t$$

2. Special space translations

$$t' = t\frac{\sqrt{1-\frac{\rho^2}{R^2}}}{1-\frac{(\mathbf{r}\boldsymbol{\rho})}{R^2}}; \qquad \mathbf{r}'_{\parallel} = \frac{\mathbf{r}_{\parallel} - \boldsymbol{\rho}}{1-\frac{(\mathbf{r}\boldsymbol{\rho})}{R^2}}$$

3. Inverse time translations

$$t' = \frac{t}{1+\frac{\tau t}{t_0{}^2}}; \qquad \mathbf{r}' = \frac{\mathbf{r}}{1+\frac{\tau t}{t_0{}^2}},$$

R-relativistic mechanics

Action for free particle with mass *m*

$$S = -\int \sum m \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

R-relativistic mechanics

Action for free particle with mass m

$$S = -\int \sum m \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

$$t = t_0 + \tau; \quad \tau << t_0; \quad |\mathbf{r}| << R$$

R-relativistic mechanics

Action for free particle with mass *m*

$$S = -\int \sum m \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

$$t = t_0 + \tau; \qquad \tau << t_0; \qquad \frac{R}{t_0} \equiv c_0; \qquad |\mathbf{r}| << R$$

$$S = -\int \sum m {c_0}^2 \sqrt{1 - \frac{\mathbf{v}^2}{{c_0}^2}} dt$$

Conserved quantities for R-space free particles

$$S = -\int \sum m \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}\, dt$$

$$H = \sum \frac{mR^2}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}}$$

$$\mathbf{K} = \sum \frac{m(\mathbf{v}t - \mathbf{r})}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}}$$

$$\mathbf{J} = \sum \frac{m(\mathbf{r} \times \mathbf{v})}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}}$$

$$\mathbf{P} = \sum \frac{m\mathbf{v}}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}}$$

$$h^2 - \mathbf{k}^2 R^2 = m^2 R^4$$

Energy in noncosmological limit

$$H = \sum \frac{mR^2}{\sqrt{1 - \dfrac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}} \qquad |\mathbf{v}t - \mathbf{r}| << R$$

$$H = \sum mR^2 (1 + \frac{(\mathbf{v}t - \mathbf{r})^2}{2R^2} - ...)$$

$$H = \sum mR^2 + \sum m \frac{(\mathbf{v}t - \mathbf{r})^2}{2}$$

$$E = \sum m \frac{\mathbf{v}^2}{2}; \qquad I(0) = \sum m \frac{(\mathbf{v}\tau - \mathbf{r})^2}{2}; \qquad F(0) = \sum m\mathbf{v}(\mathbf{r} - \mathbf{v}\tau).$$

Energy in noncosmological limit

$$H = \sum \frac{mR^2}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}}$$

$$\left|\mathbf{v}t_0 - \mathbf{r}\right| << R$$

$$t = t_0 + \tau; \qquad \tau << t_0$$

$$H = \sum mR^2 (1 + \frac{(\mathbf{v}(t_0 + \tau) - \mathbf{r})^2}{2R^2} - ...)$$

$$H = \sum mR^2 + \sum m \frac{(\mathbf{v}\tau - \mathbf{r})^2}{2} + t_0 \sum m\mathbf{v}(\mathbf{v}\tau - \mathbf{r}) + t_0{}^2 \sum m \frac{\mathbf{v}^2}{2}$$

$$E = \sum m \frac{\mathbf{v}^2}{2}; \qquad I(0) = \sum m \frac{(\mathbf{v}\tau - \mathbf{r})^2}{2}; \qquad F(0) = \sum m\mathbf{v}(\mathbf{r} - \mathbf{v}\tau).$$

Anti-de Sitter Action in Beltrami coordinates

$$S = -\int \sum \frac{mc^2 R^2}{\left(R^2 + c^2 t^2 - \mathbf{r}^2\right)} \sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}} \; dt.$$

$$\frac{1}{{c_0}^2} = \frac{1}{c^2} + \frac{{t_0}^2}{R^2}$$

Nonrelativistic limit

$$c \to \infty$$

$$S = -\int \sum m \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

Noncosmological limit

$$R \to \infty$$

$$S = -\int \sum mc^2 \sqrt{1 - \frac{\mathbf{v}^2}{c^2}} dt$$

Combined (Anti-de Sitter) deformation of Galilean symmetry

$$ct' = \frac{ct - \frac{\mathbf{vr}}{c}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}; \qquad \mathbf{r}_{\parallel}' = \frac{\mathbf{r}_{\parallel} - \mathbf{v}t}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}};$$

Lorentz

$$t' = \frac{t\sqrt{1 - \frac{\rho^2}{R^2}}}{1 - \frac{(\mathbf{r}\boldsymbol{\rho})}{R^2}}; \qquad \mathbf{r}'_{\parallel} = \frac{\mathbf{r}_{\parallel} - \boldsymbol{\rho}}{1 - \frac{(\mathbf{r}\boldsymbol{\rho})}{R^2}},$$

Space translations

$$t' = \frac{t - \theta}{1 + \frac{c^2\theta t}{R^2}}; \qquad \mathbf{r}' = \frac{\mathbf{r}\sqrt{1 + \frac{c^2\theta^2}{R^2}}}{1 + \frac{c^2\theta t}{R^2}},$$

Time translations

Conserved quantities for Anti-de Sitter free particles

$$H = \sum \frac{m}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}}; \qquad \mathbf{P} = \sum \frac{m\mathbf{v}}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}};$$

$$\mathbf{K} = \sum \frac{m(\mathbf{v}t - \mathbf{r})}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}}; \qquad \mathbf{J} = \sum \frac{m[\mathbf{v} \times (\mathbf{v}t - \mathbf{r})]}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}}$$

$$h^2 - \frac{\mathbf{k}^2}{R^2} - \frac{\mathbf{p}^2}{c^2} + \frac{\mathbf{j}^2}{R^2 c^2} = m^2 \qquad \mathbf{j} = \frac{[\mathbf{p} \times \mathbf{k}]}{h}$$

“Two” Galilean relativity are identical

$$S = -\int \sum \frac{mc^2 R^2}{\left(R^2 + c^2 t^2 - \mathbf{r}^2\right)} \sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}} \; dt.$$

$c \to \infty$

$$S = -\int \sum m \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

$R \to \infty$

$$S = \int \sum \frac{m(\mathbf{v}t - \mathbf{r})^2}{2t^2} dt$$

$R \to \infty$

$$S = -\int \sum mc^2 \sqrt{1 - \frac{\mathbf{v}^2}{c^2}} dt$$

$c \to \infty$

$$S = \int \sum \frac{m\mathbf{v}^2}{2} dt$$

Nonrelativistic limit $c \to \infty$

$$ct >> R, \quad t >> \frac{R}{c} = \tau ?$$

$$\tau = \sqrt{\frac{\hbar G_N}{c^5}} = 5.39 \cdot 10^{-44}\,\text{c}$$

$$t_0 = 10^{60}\,\tau$$