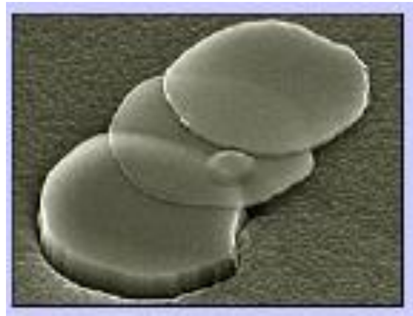
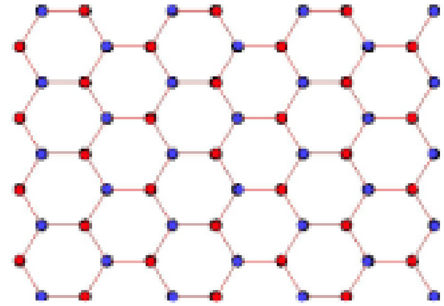
Graphene Physics

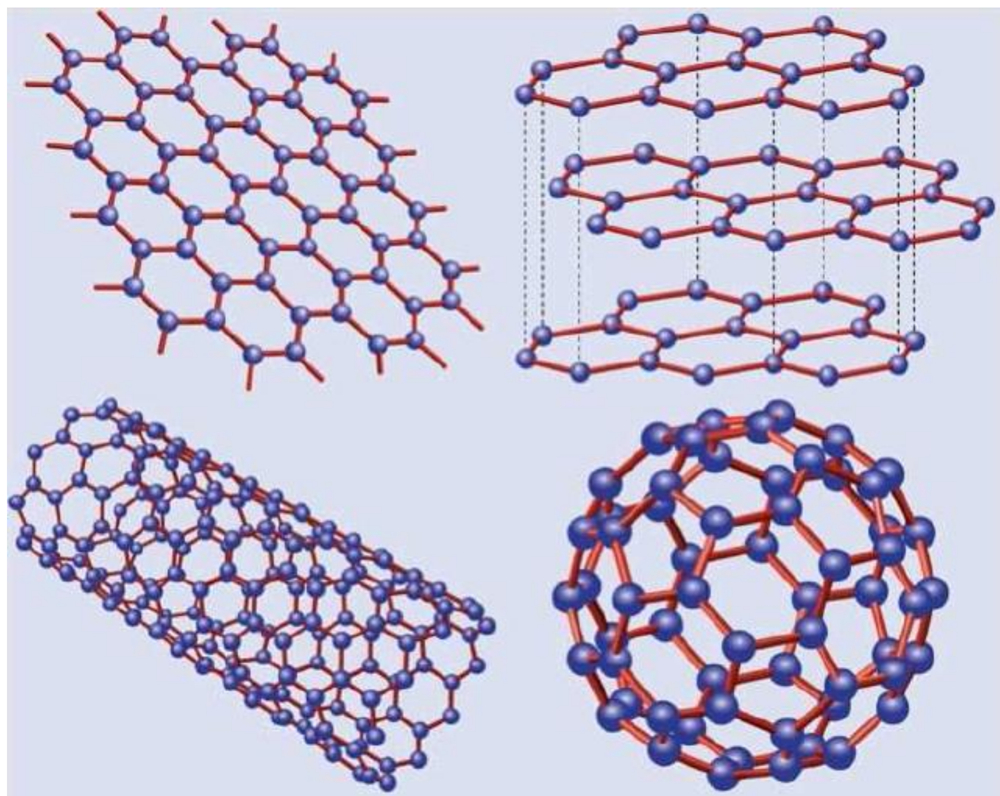
L.A. Falkovsky
Landau Institute for Theoretical Physics

outline

- electron dispersion in graphene, graphene bilayer and graphite
- conductivity in optical region
- universal dynamical conductivity of graphene
- Kerr effect and reflectivity in magnetic field

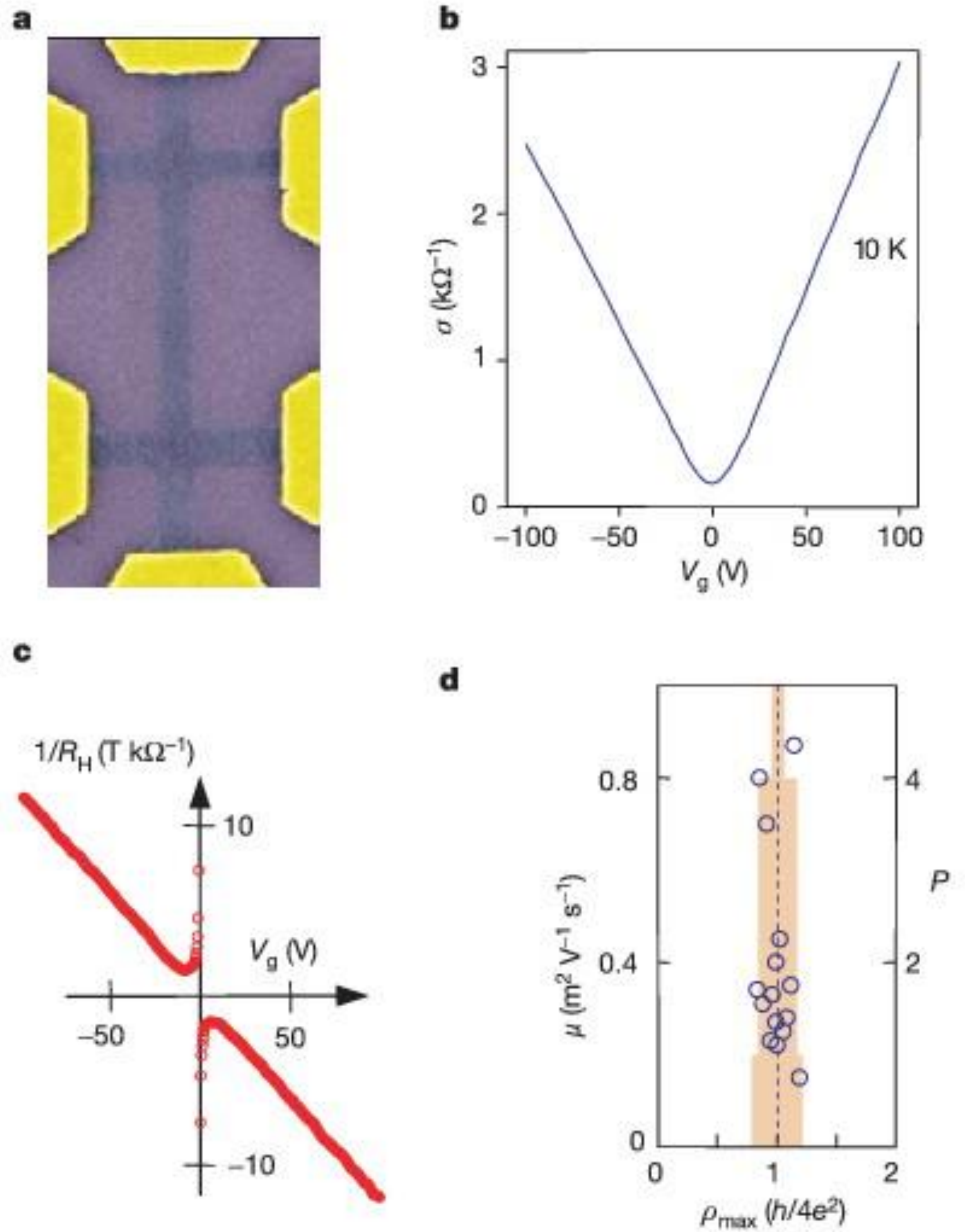






Field-effect

Novoselov et al (2005)



1) conductivity proportional to the carrier density

2) universal minimal conductivity

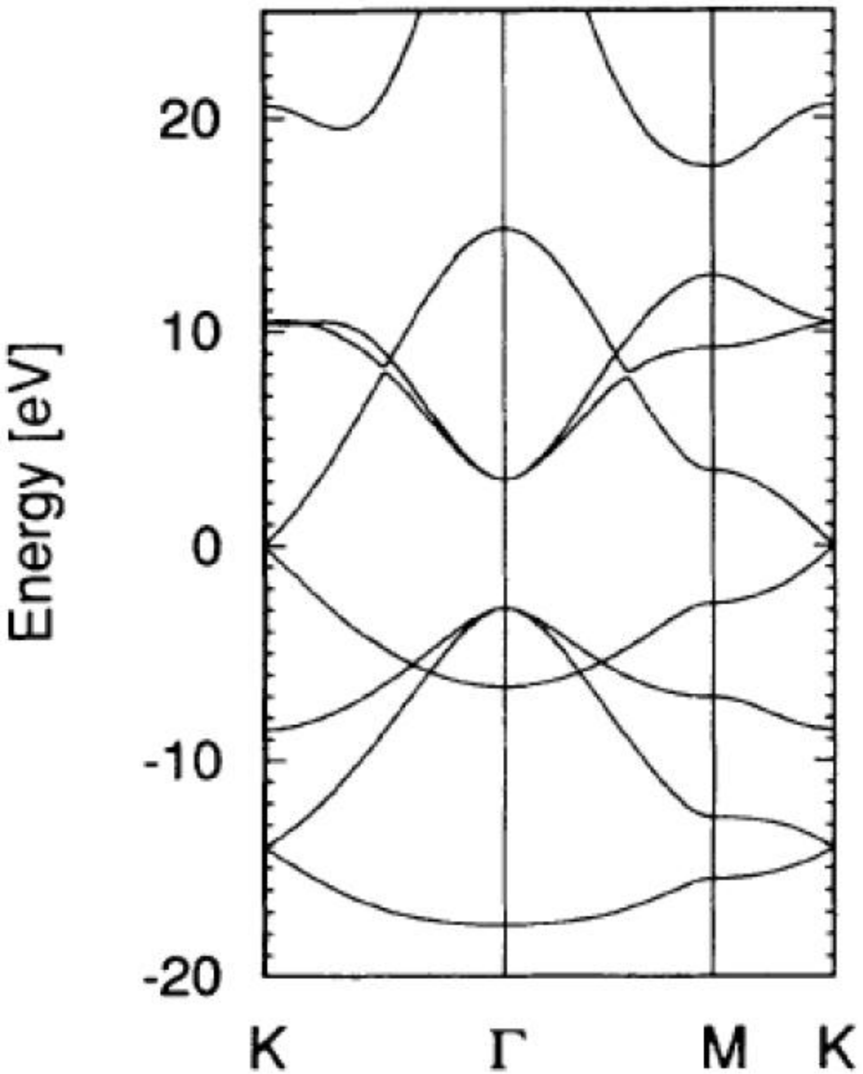
$$\sigma_{\min} = \frac{2e^2}{\pi\hbar}$$

3) universal optical conductivity

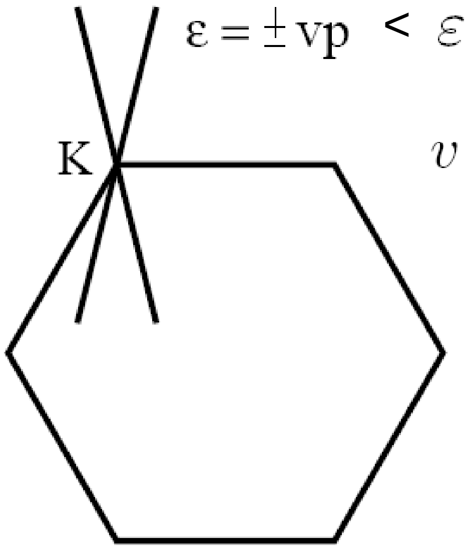
$$\sigma_{opt}(\omega) = \frac{e^2}{4\hbar}$$

Electron bands

graphene



two bands of opposite parity at K



$\varepsilon = \pm vp < \varepsilon_{at} \simeq 5 \text{ eV}$

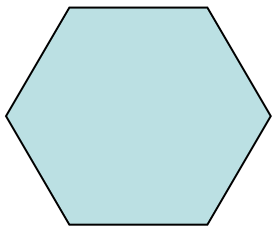
$v = 10^8 \text{ cm/s}$

$\varepsilon_g \sim 1 \text{ K}$

Brillouin zone

Electron dispersion

Wallace (1947)
Slonczewski and Weis (1955)



at the K point in the Brillouin zone

two-dimensional representation of the small group C_{3V}

$$C_3 \psi_{\mp} = e^{\mp \frac{2\pi i}{3}} \psi_{\mp}$$

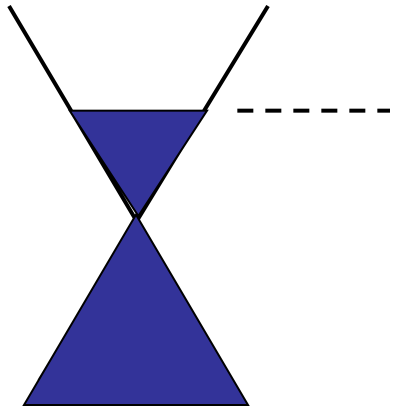
$$C_3 (p_x \mp i p_y) = e^{\mp \frac{2\pi i}{3}} (p_x \mp i p_y)$$

$$\begin{array}{c|cc|c} & \psi_+ & \psi_- & \\ \hline H = v & 0 & p_- & \psi_+^* \\ & p_+ & 0 & \psi_-^* \end{array}$$

$$\varepsilon_{1,2} = \pm v \sqrt{p_x^2 + p_y^2}$$

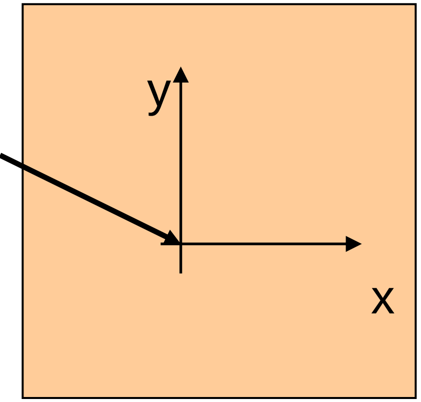
two-dimensional massless Dirac fermions

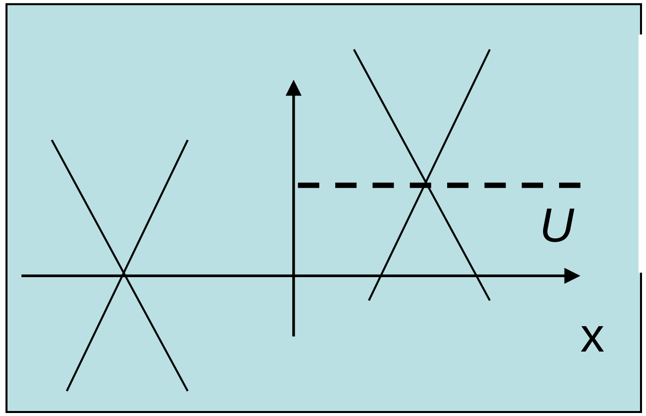
$$vp \; ? \; \frac{e^2}{r}$$



graphene doped

Klein's paradox





X>0

$$H(\mathbf{p}) = v \begin{pmatrix} U & ip_x - p_y \\ -ip_x - p_y & U \end{pmatrix}$$

X<0 *U=0*

X<0

$$\psi = \exp(ip_x x) \begin{pmatrix} ivp_+/\sqrt{\varepsilon^2 + v^2p^2} \\ \varepsilon/\sqrt{\varepsilon^2 + v^2p^2} \end{pmatrix} + C_1 \exp(-ip_x x) \begin{pmatrix} -ivp_-/\sqrt{\varepsilon^2 + v^2p^2} \\ \varepsilon/\sqrt{\varepsilon^2 + v^2p^2} \end{pmatrix}$$

X>0

$$\psi = C_2 \exp(i\tilde{p}_x x) \begin{pmatrix} iv\tilde{p}_+/\sqrt{(\varepsilon - U)^2 + v^2\tilde{p}^2} \\ (\varepsilon - U)/\sqrt{(\varepsilon - U)^2 + v^2\tilde{p}^2} \end{pmatrix}$$

$$\tilde{p}_+ = \sqrt{[(\varepsilon - U)/v]^2 - p_y^2} + ip_y$$

$$p_+ = \sqrt{(\varepsilon/v)^2 - p_y^2} + ip_y$$

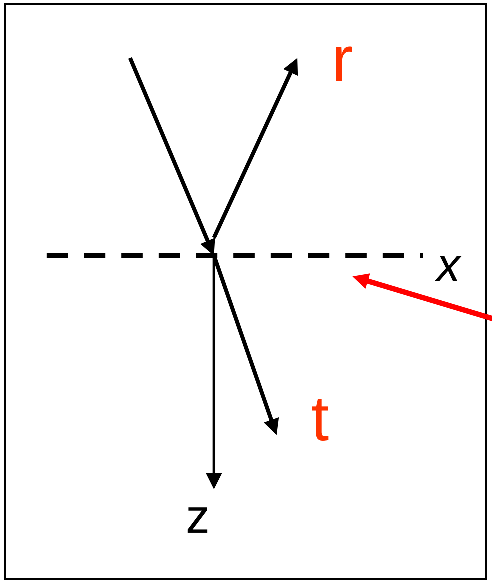
Graphene light transmittance

Kuzmenko et al

Novoselov et al

Basov et al

$$T = 1 - ?$$



suspended graphene

conductivity in collisionless limit

Falkovsky, Varlamov

$$j_i(x) = e\tilde{\psi}^+(x')v^i_{x'x}\tilde{\psi}(x) - \frac{e^2}{c}\tilde{\psi}^+(x')(m^{-1})^{ij}_{x'x}\tilde{\psi}(x)A_j$$

$$V = -\frac{e}{c}\int \psi^+(x')v^i_{x'x}\psi(x)A_i(x)d^{d+1}x$$

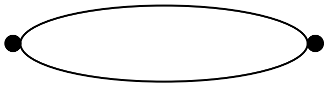
$$\mathcal{P}(\omega_l, \mathbf{k}) = T\sum_{\mathbf{p}\,\omega_n} Tr\left\{v^i\mathcal{G}(p_+)\,v^j\mathcal{G}(p_-)\right\}$$

$$\mathcal{G}(p) = [i\omega_n - H(\mathbf{p})]^{-1}$$

$$p_\pm = (\omega_n \pm \omega_l/2, \mathbf{p} \pm \mathbf{k}/2)$$

$$Tr\left\{v^i\mathcal{G}v^j\mathcal{G}\right\} = v^i_{11}\mathcal{G}_{11}v^j_{11}\mathcal{G}_{11} + v^i_{22}\mathcal{G}_{22}v^j_{22}\mathcal{G}_{22} + v^i_{12}\mathcal{G}_{22}v^j_{21}\mathcal{G}_{11} + v^i_{21}\mathcal{G}_{11}v^j_{12}\mathcal{G}_{22}$$

$$\mathbf{v} = \partial H/\partial\mathbf{p}$$



$$T\sum_{\omega_n}\mathcal{G}_{11}(p_+)\mathcal{G}_{22}(p_-) = \frac{f[\varepsilon_1(\mathbf{p}_-)] - f[\varepsilon_2(\mathbf{p}_+)]}{i\omega_l - \varepsilon_2(\mathbf{p}_+) + \varepsilon_1(\mathbf{p}_-)}$$

$$\omega_n = 2\pi T(n + 1/2)$$

$$\sigma_{ij}(\omega, k) = \frac{ie^2}{\pi^2}\times$$

$$\mathrm{p}_\pm = \mathrm{p} \pm \frac{\mathrm{k}}{2}$$

$$\left\{\sum_{a=1,2}\int\frac{d^2pv^iv^j\{f_0[\varepsilon_a(\mathbf{p}_-)] - f_0[\varepsilon_a(\mathbf{p}_+)]\}}{[\varepsilon_a(\mathbf{p}_+) - \varepsilon_a(\mathbf{p}_-)][\omega - \varepsilon_a(\mathbf{p}_+) + \varepsilon_a(\mathbf{p}_-)]}\right.$$

$$\to -\frac{df[\varepsilon_a(\mathbf{p})]}{d\varepsilon}\frac{1}{\omega - \mathbf{kv}}$$

$$\left. + 2\omega\int\frac{d^2pv^i_{12}v^j_{21}\{f_0[\varepsilon_1(\mathbf{p}_-)] - f_0[\varepsilon_2(\mathbf{p}_+)]\}}{[\varepsilon_2(\mathbf{p}_+) - \varepsilon_1(\mathbf{p}_-)]\{\omega^2 - [\varepsilon_2(\mathbf{p}_+) - \varepsilon_1(\mathbf{p}_-)]^2\}}\right\}$$

$$\omega \to \omega + i\tau^{-1}$$

optical conductivity

Falkovsky, Pershoguba

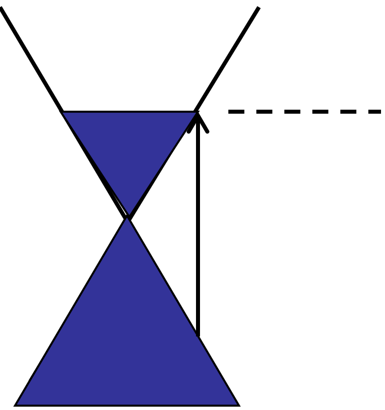
Gusynin , Sharapov

$$\omega \gg (kv, \tau^{-1})$$

integrated over angle

$$\sigma(\omega) = \frac{e^2\omega}{i\pi\hbar}\left[\int_{-\infty}^{+\infty} d\varepsilon \frac{|\varepsilon|}{\omega^2}\frac{df(\varepsilon)}{d\varepsilon} - \int_0^{+\infty} d\varepsilon \frac{f(-\varepsilon)-f(\varepsilon)}{(\omega+i\delta)^2-4\varepsilon^2}\right]$$

$$\sigma^{intra}(\omega) = \frac{2ie^2T}{\pi\hbar(\omega+i\tau^{-1})}\ln\left[2\cosh(\mu/2T)\right]$$

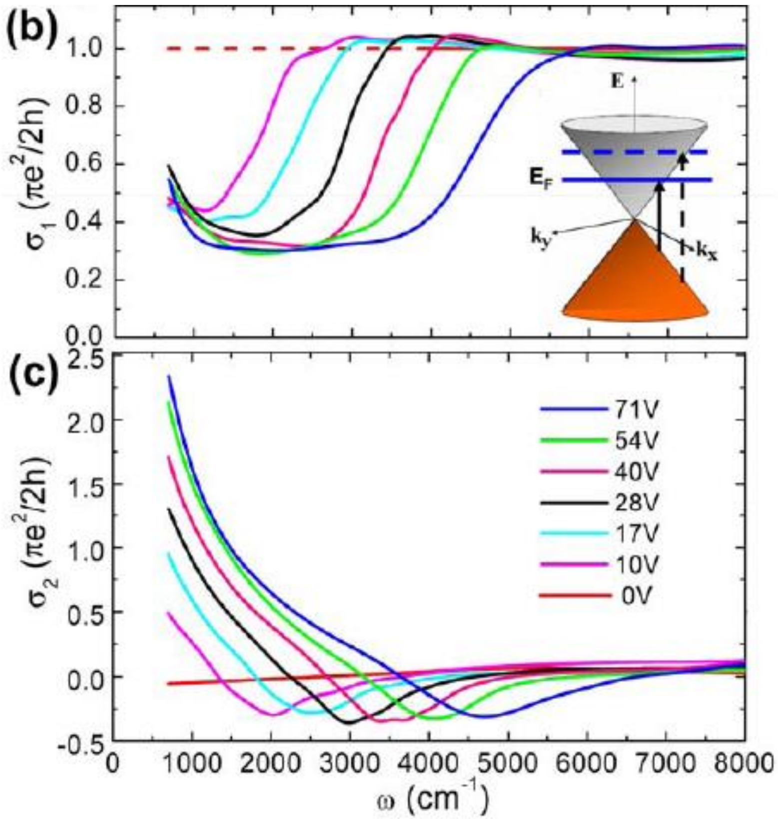


$$\sigma^{inter}(\omega) = \frac{e^2}{4\hbar}\left[\theta(\omega-2\mu) - \frac{i}{2\pi}\ln\frac{(\omega+2\mu)^2}{(\omega-2\mu)^2}\right] \qquad T=0$$

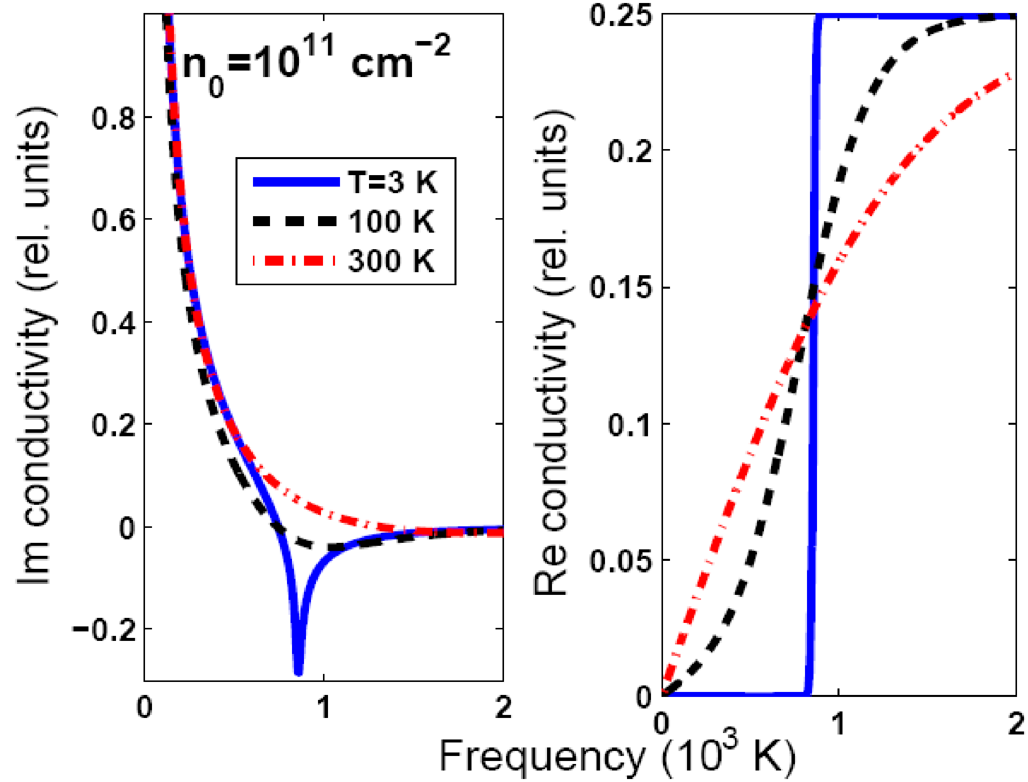
$$\theta(\omega-2\mu) \rightarrow \frac{1}{2} + \frac{1}{\pi}\arctan[(\omega-2\mu)/2T] \qquad T \ll \mu$$

$$(\omega-2\mu)^2 \rightarrow (\omega-2\mu)^2 + (2T)^2 \,.$$

graphene conductance

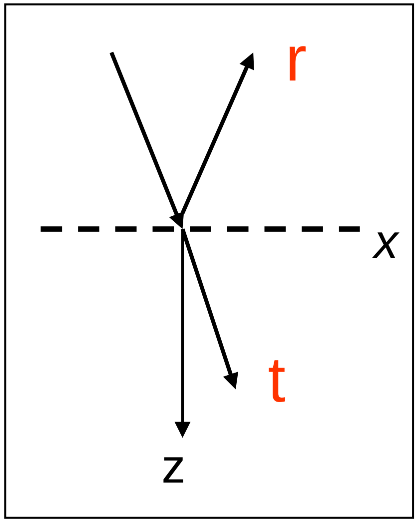


$$e^2/\hbar$$



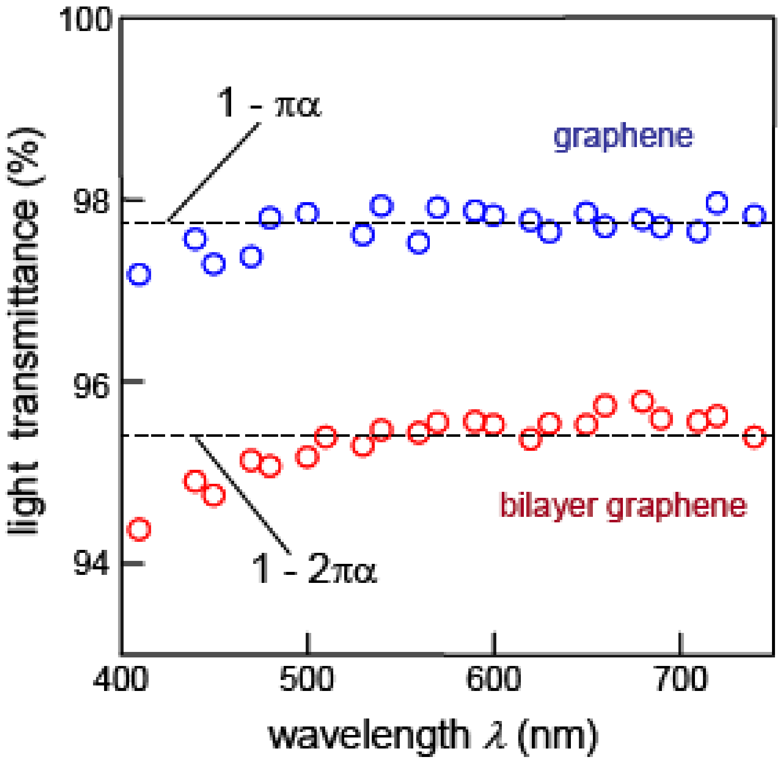
D. Basov et al

graphene transmittance



$$T = 1 - \pi \frac{e^2}{\hbar c}$$

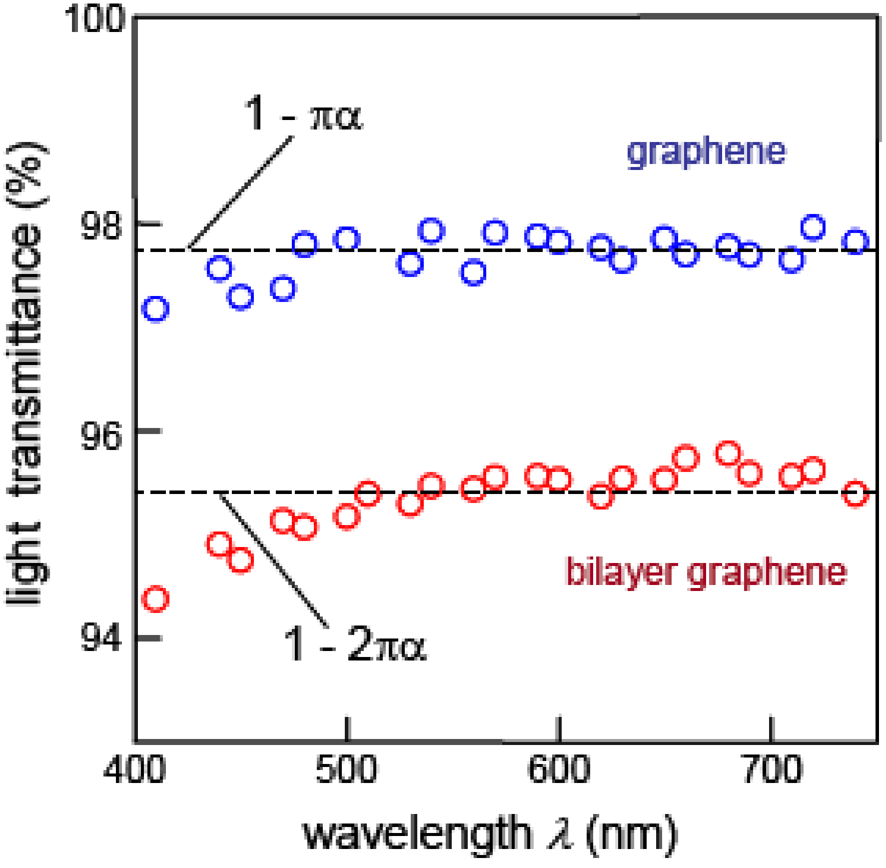
normal incidence

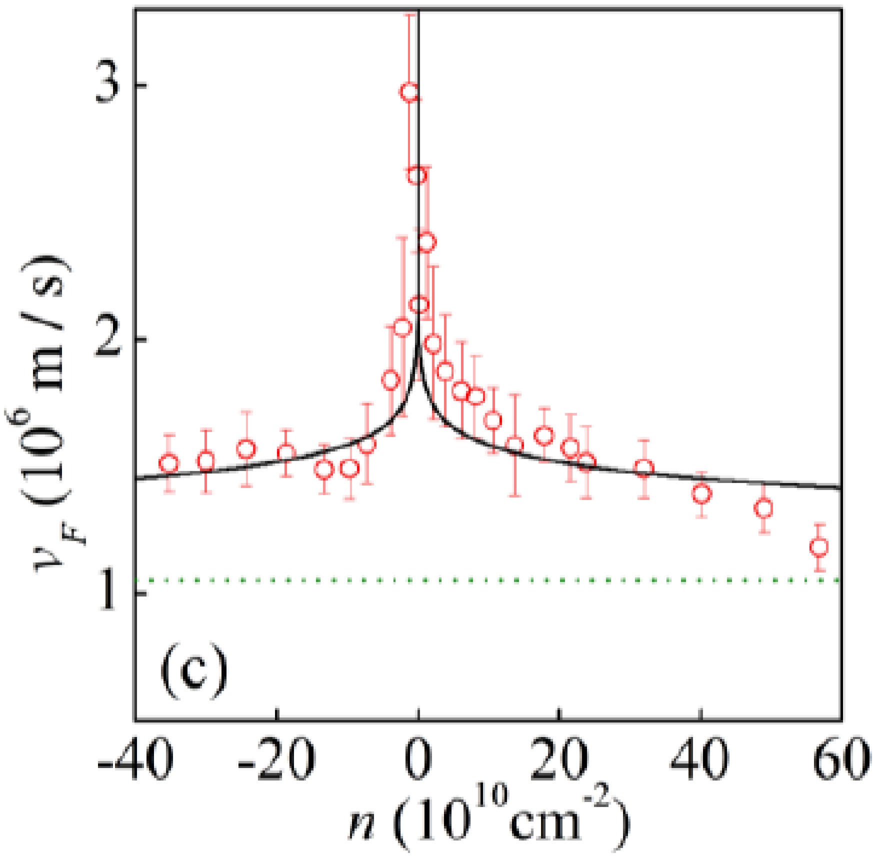


1) fine structure constant
2) no material parameters
3) no effect of e-e interactions

Li et al,
Nair et al,
Mak et al

Renormalization of the Fermi velocity due to el-el interactions





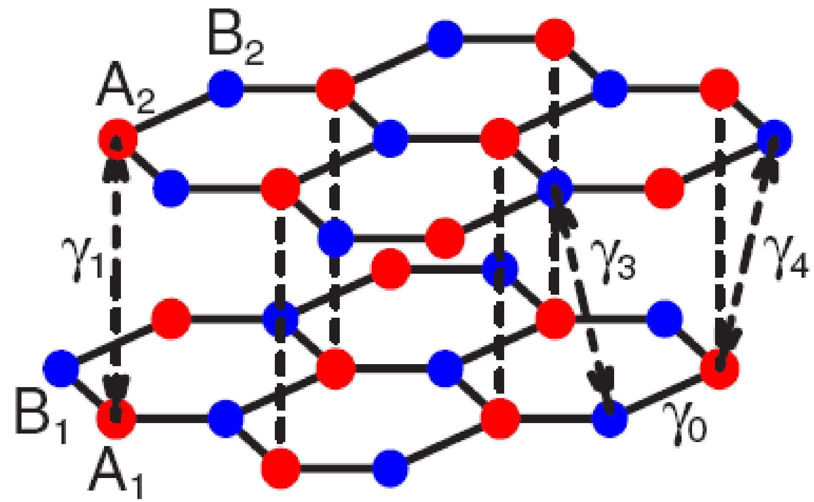
Graphene bilayer in the tight-binding approximation

$$\psi_a = \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}\mathbf{a}_j} \psi_0(\mathbf{a}_j - \mathbf{r})$$

$$\psi_b = \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}\mathbf{a}_j} \psi_0(\mathbf{a}_j + \mathbf{a} - \mathbf{r})$$

$$\psi_{a1} = \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}\mathbf{a}_j} \psi_0(\mathbf{a}_j + \mathbf{c} - \mathbf{r})$$

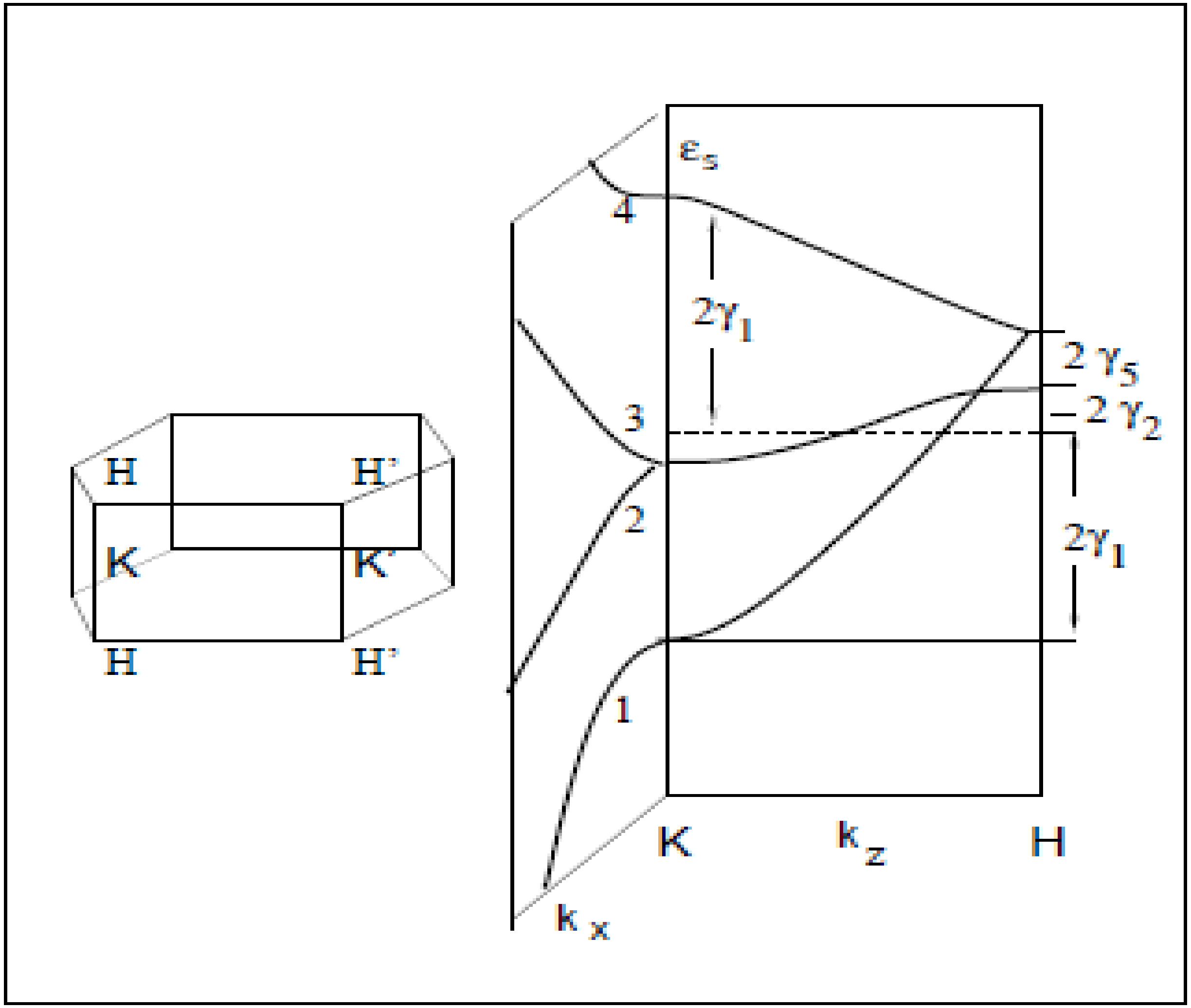
$$\psi_{b1} = \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}\mathbf{a}_j} \psi_0(\mathbf{a}_j + \mathbf{c} + \mathbf{a} - \mathbf{r}),$$



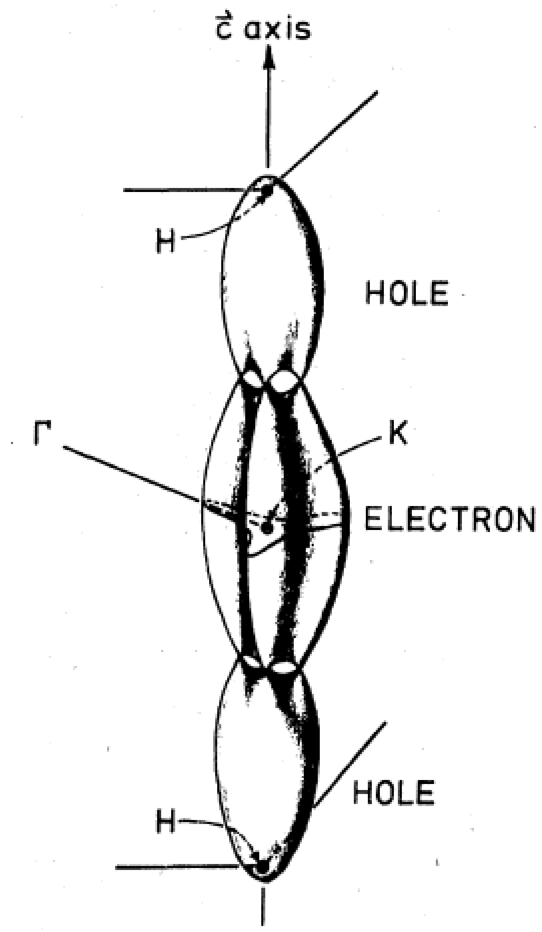
Effective Hamiltonian in a simplest form

$$H(\mathbf{k}) = \begin{pmatrix} U & vk_+ & \gamma_1 & 0 \\ vk_- & U & 0 & 0 \\ \gamma_1 & 0 & -U & vk_- \\ 0 & 0 & vk_+ & -U \end{pmatrix}$$

Brillouin zone and electronic bands of graphite



Fermi surface in graphite



Hamiltonian for graphite

$$H(\mathbf{k}) = \begin{pmatrix} \tilde{\gamma}_5 & vk_+ & \tilde{\gamma}_1 & \tilde{\gamma}_4 vk_-/\gamma_0 \\ vk_- & \tilde{\gamma}_2 & \tilde{\gamma}_4 vk_-/\gamma_0 & \tilde{\gamma}_3 vk_+/\gamma_0 \\ \tilde{\gamma}_1 & \tilde{\gamma}_4 vk_+/\gamma_0 & \tilde{\gamma}_5 & vk_- \\ \tilde{\gamma}_4 vk_+/\gamma_0 & \tilde{\gamma}_3 vk_-/\gamma_0 & vk_+ & \tilde{\gamma}_2 \end{pmatrix}$$

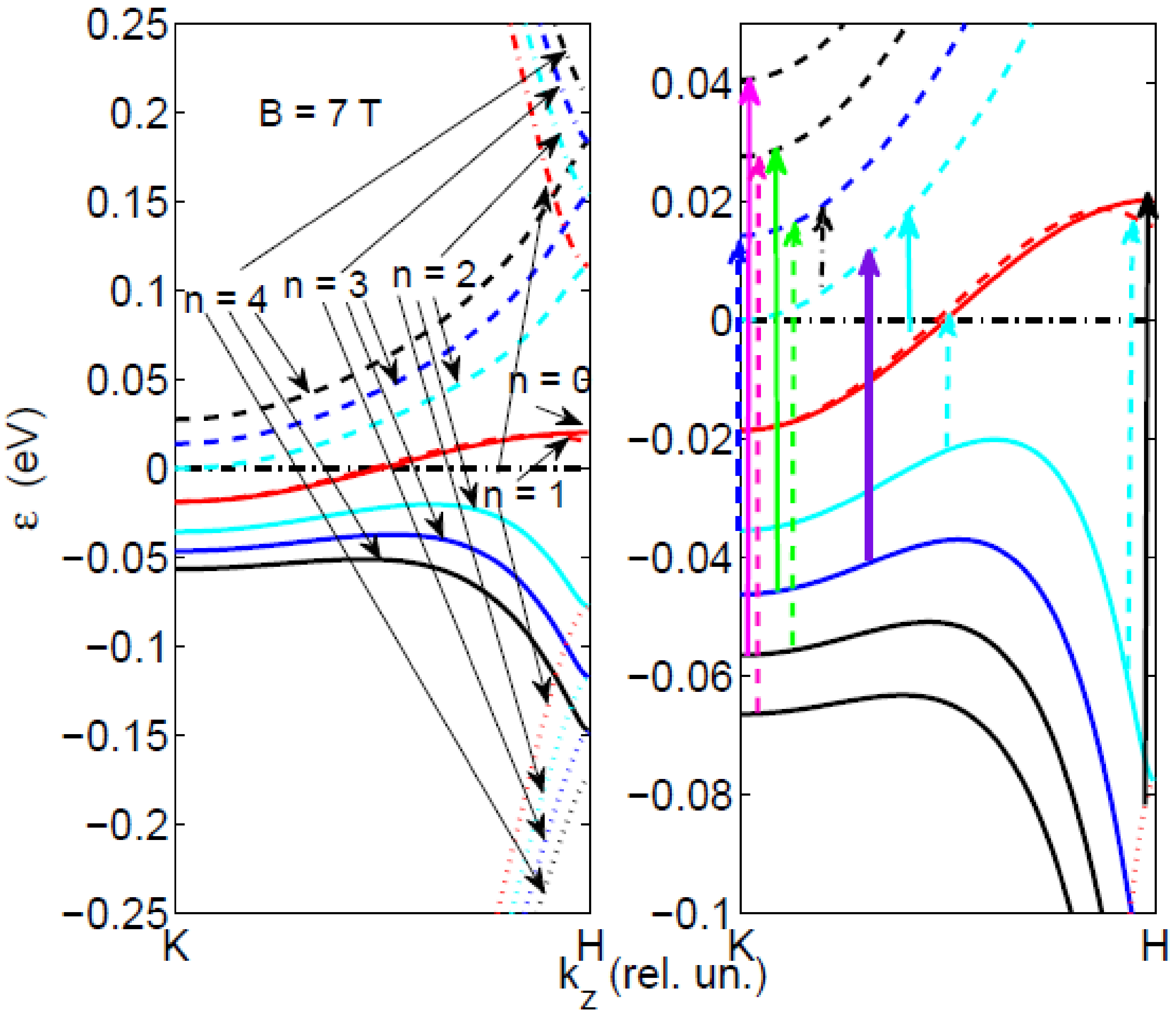
in magnetic field

$$\hat{k}_+ = \sqrt{2|e|\hbar B/c}\, a, \quad \hat{k}_- = \sqrt{2|e|\hbar B/c}\, a^+$$

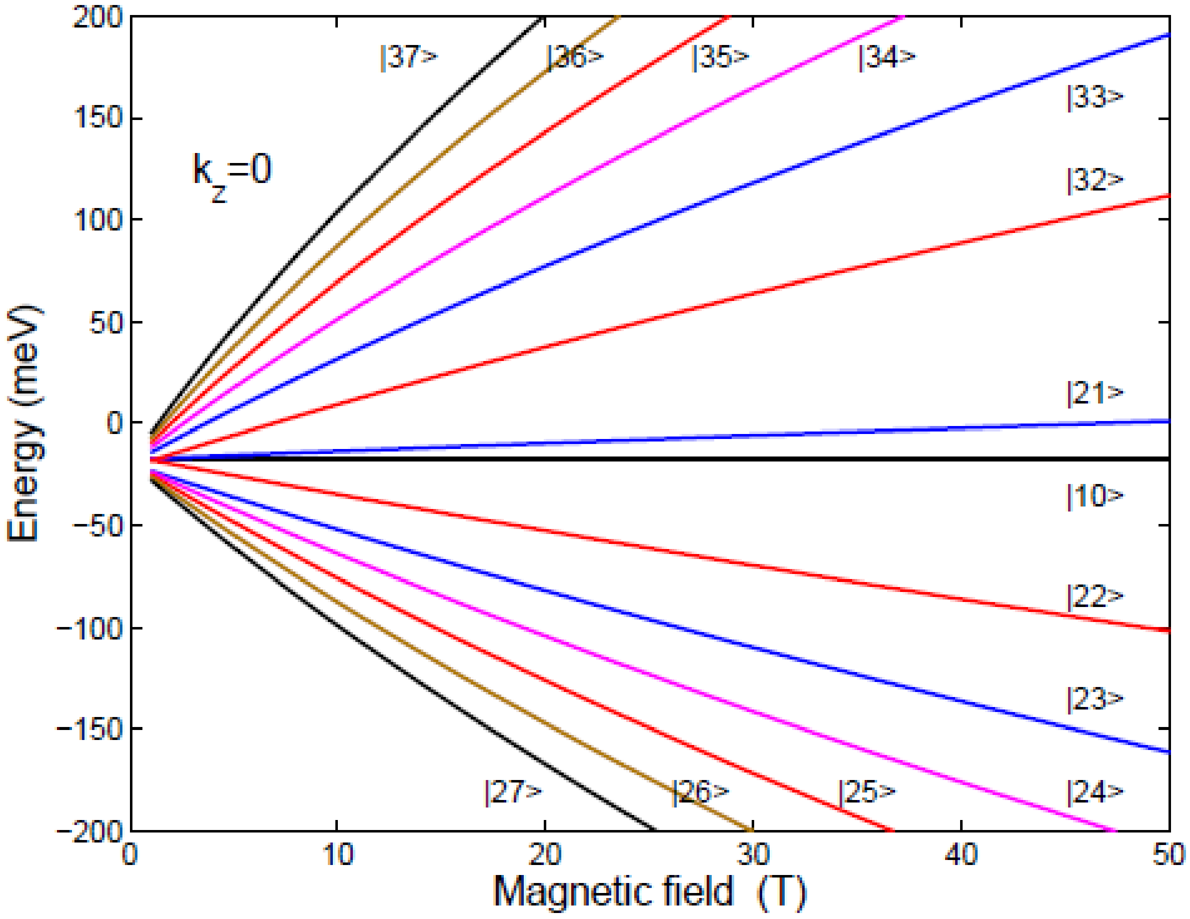
wave function

$$\psi^{\alpha}_{sn}(x) = \begin{cases} C^1_{sn}\varphi_{n-1}(x) \\ C^2_{sn}\varphi_n(x) \\ C^3_{sn}\varphi_{n-1}(x) \\ C^4_{sn}\varphi_{n-2}(x) \end{cases}$$

Band dispersion in magnetic field



Nearest Landau levels versus magnetic field



Conductivities in the main approximation

$$\left.\begin{array}{c}\sigma_{xx}(\omega)\\ i\sigma_{xy}(\omega)\end{array}\right\} = i\sigma_0 \frac{4\omega_c^2}{\pi^2} \sum_{n,s,s'} \int_0^{\pi/2} dz \frac{\Delta f_{ss'n}}{\Delta ss'n} |d_{ss'n}|^2$$

$$\times \left[(\omega + i\Gamma + \Delta_{ss'n})^{-1} \pm (\omega + i\Gamma - \Delta_{ss'n})^{-1} \right],$$

$$\Delta f_{ss'n} = f(\varepsilon_{s'n+1}) - f(\varepsilon_{sn})$$

$$\Delta_{ss'n} = \varepsilon_{sn} - \varepsilon_{s',n+1}$$

$$\omega_c = v\sqrt{2|e|\hbar B/c} \qquad \sigma_0 = \frac{e^2}{4\hbar c_0}$$

$$d_{ss'n} = C_{sn}^2 C_{s'n+1}^{1*} + C_{sn}^3 C_{s'n+1}^{4*}$$
$$+(\tilde{\gamma}_4/\gamma_0)(C_{sn}^1 C_{s'n+1}^{4*} + C_{sn}^2 C_{s'n+1}^{3*})$$

Topological confinement in bilayer graphene

zero-modes are chiral, they have the definite sign of $s_y p_y$

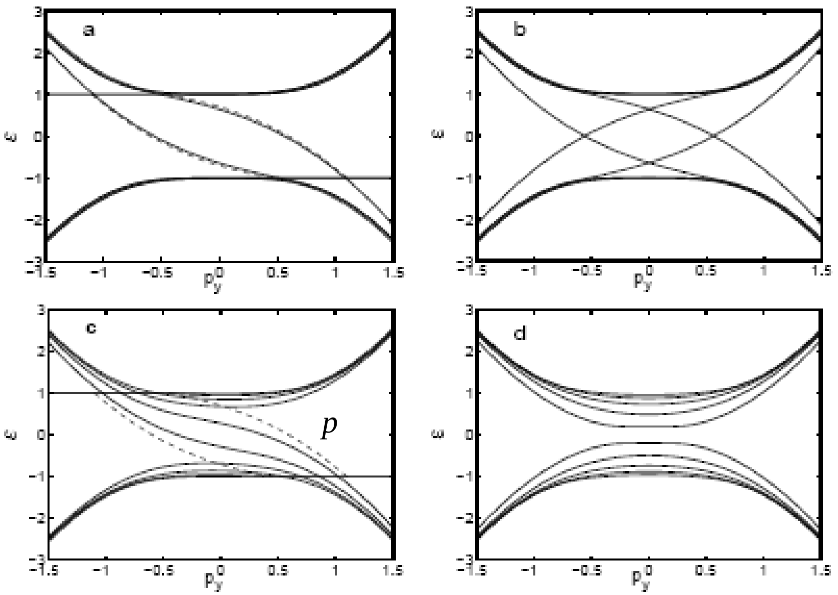


FIG. 3: Energy level structure in the presence of non-uniform interlayer bias $\varphi(x)$. The low-energy states in (a) and (c) are localized on the single kink of profile $\varphi(x) = \tanh(x/\ell)$ with $\ell = 0.5$ and $\ell = 4$, respectively. For (a) and (c) we used open boundary conditions, $\Psi(\pm 20) = 0$, which caused flat bands at $\varepsilon = \pm 1$. The dashed lines are the analytical expressions for the intragap state dispersion, Eq. (14). (b): Same as (a) but with kink and anti-kink at $x = \pm 10$ and periodic boundary conditions. (d): Non-topological confinement for potential profile $\varphi(x) = \tanh^2(x/4)$. Note the absence of chiral zero-modes.

Martin et al (2007)

Conclusions

- in-layer graphite conductivity per layer is close to the graphene conductivity
- it contains singularity at the inter-layer hopping energy
- anisotropy of dynamic conductivity is on the order of 100
- Kerr rotation in graphite is on the order of 15 degrees in fields 50 T

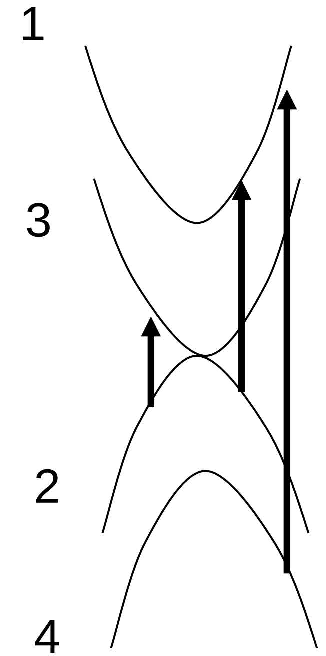
Contributions of electron transitions in conductivity

$$\gamma_2, \gamma_5 < \omega < \gamma_0$$

$$\mathrm{Re}\ \sigma_{23} = \frac{e^2}{4\pi\hbar c_0} \int_0^{\pi/2} dz \frac{2\gamma(z) + \omega}{\gamma(z) + \omega},$$

$$\mathrm{Re}\ \sigma_{21} = \frac{e^2}{4\pi\hbar c_0} \int_0^{\pi/2} dz \frac{\gamma^2(z)}{\omega^2} \theta[\omega - \gamma(z)],$$

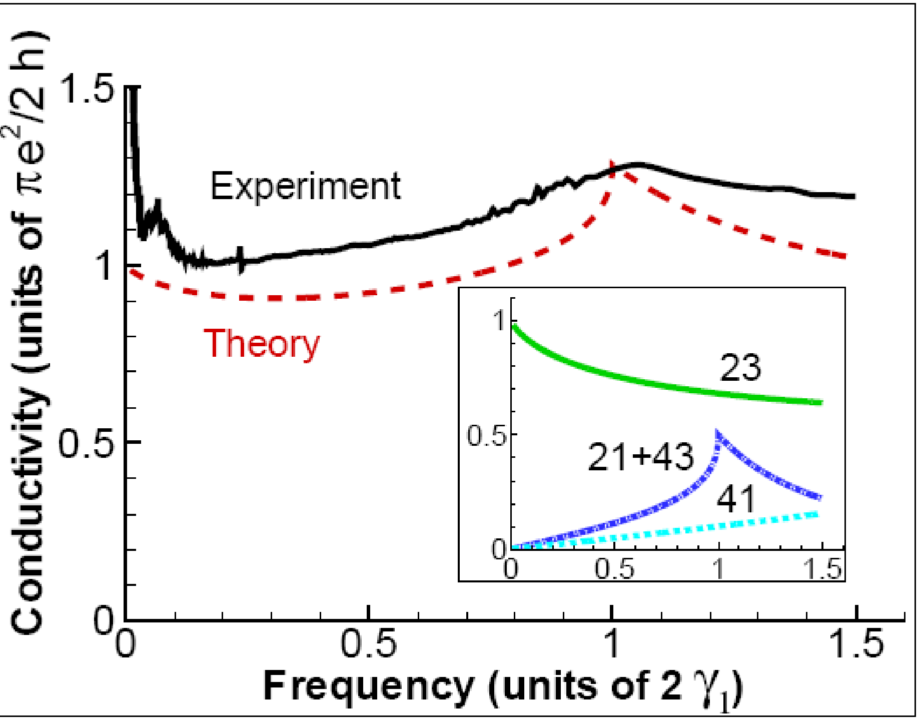
$$\mathrm{Re}\ \sigma_{41} = \frac{e^2}{4\pi\hbar c_0} \int_0^{\pi/2} dz \frac{2\gamma(z) - \omega}{\gamma(z) - \omega} \theta[\omega - 2\gamma(z)]$$

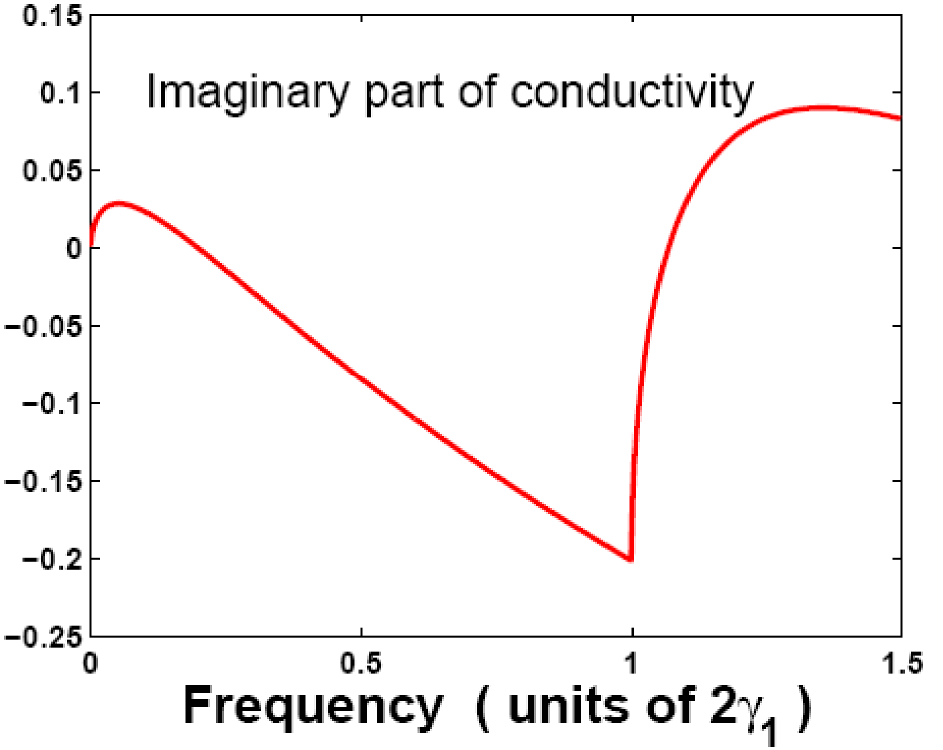


$$\gamma(z) = 2\gamma_1 \cos z \qquad \sigma_0 = \frac{e^2}{4\hbar c_0}$$

Real and imaginary parts of conductivity in graphite

in units of $$\sigma_0 = \frac{e^2}{4\hbar c_0}$$





experiment: Kuzmenko et al (2009)
theory: Falkovsky (2010)

zz conductivity

$$v_z = \frac{\partial \varepsilon_3}{\partial k_z} \sim \gamma_1 c_0 \sin(k_z c_0)$$

$$\sigma_{zz}/\sigma_0 \sim \left(\gamma_1/\gamma_0\right)^2$$

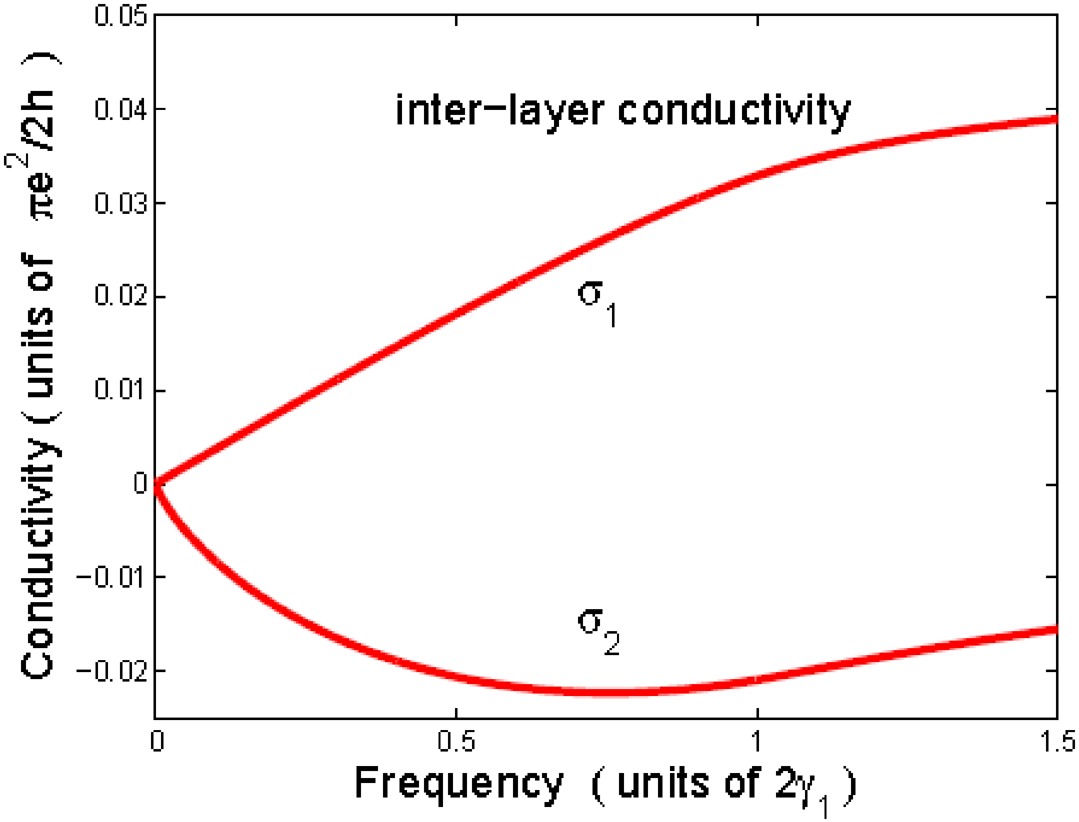
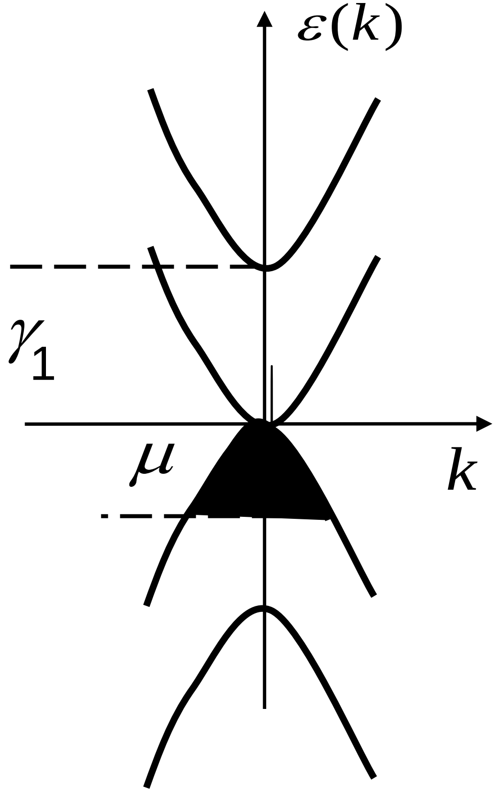


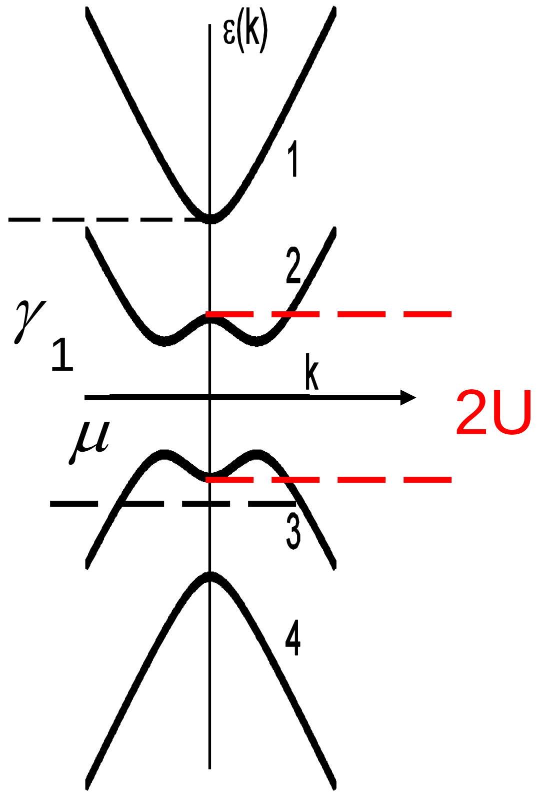
Table 3: Top 10 most productive countries of authors dealing with graphene. Source: CAplus on STN Internat

#	Author Country	# Articles	% Articles
1	USA	1106	30.6
2	Japan	514	14.2
3	Germany	215	6.0
4	Peop. Rep. China	199	5.5
5	France	181	5.0
6	UK	144	4.0
7	Spain	137	3.8
8	Netherlands	99	2.7
9	Russia	96	2.7
10	Italy	90	2.5

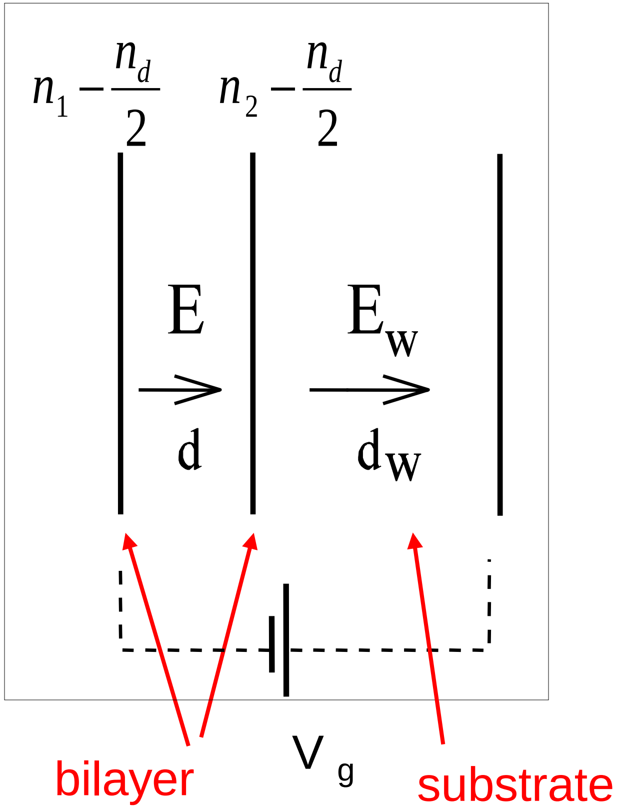
Bilayer electron dispersion



γ_1 = 0.4 eV



Gap 2U in external field E=2U/d



Schematic electrostatic model

d = 3.35 Å

d << d $_W$

Lu et al
McCann, Fal'ko
Gava et al
Ohta et al
Kuzmenko et al
Mak et al