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Neutron Multiwave Spin Echo

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Abstracts—The neutron multiwave interference mode is investigated using the spin echo technique. In this mode a neutron wave repeatedly splits in the magnetic field of resonance coils, which results in the appearance of additional maxima of a constructive interference being absent in the well-known classical and resonance neutron spin echo modes. Simple analytical expressions well describing the experimental data are presented. It is demonstrated that the multiwave part of a spin echo signal appears when the spin flip probability in radiofrequency coils of a resonance spin echo device is $\rho < 1$. The possibility to use the neutron multiwave spin echo mode for investigation of high-order correlation functions, spatial and time correlations of three and more particles, is discussed.

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INTRODUCTION

Neutron spin echo (NSE) spectroscopy, proposed for the first time by F. Mezei in 1972 [1], was developed as the effective measuring method of neutron scattering in the experiments on the structure and dynamics investigation of condensed media. The essence of the method is recording of the scattering process by comparison of the phase of the Larmor neutron spin precession in two analogous magnetic fields arranged in two spin echo device arms before and after scattering event. It is well known that the Larmor precession is equivalent to the Zeeman split of an incident neutron wave into two waves with different wave vectors and different spin states. The phase shift accumulated between two split waves in the first spin echo arm corresponds to the distance R_{1-2} between wave fronts (or the time t_{1-2} between them). Passing through the sample and interacting with it, this neutron carries information on correlations in space at the scale R_{1-2} (or in time at the scale T_{1-2}). The second spin echo device arm compensates for the phase shift of two waves, as if the sample were absent and, consequently, separates the phase difference between the parts of a split neutron wave formed as a result of interaction with the sample (neutron wave parts transmit in the sample along the spatially separated trajectories or at different time).

In the classical type of NSE technique, spin precession appears in constant magnetic fields behind and beyond the scattering sample. Gaehler and Golub proposed a new method for obtaining spin precession by resonance coils and developed the neutron resonance spin echo (NRSE) technique [2–4]. In this spin echo modification, a polarized neutron beam transmits subsequently through two resonance coils (RCs) with magnetic field inside, consisting of the constant component B_0 and the oscillating component B_{RF}, and the field oscillation frequency B_{RF} was selected in resonance with the Larmor neutron frequency precessing in the field B_0. The precessing field amplitude is arranged so that a neutron spin in a coil is flipped under flight through an RC. Gaehler and Golub's contribution is that they demonstrated experimentally and theoretically that a neutron spin in the system consisting of two RCs placed at the distance L from each other precesses as if a neutron had passed the distance L through the field $2B_0$.

If the spin flip probability in an RC is not 100%, $\rho < 1$, each neutron wave transmitting through a coil splits into two and, as a result, the number of waves leaving an RC is doubled. The possibility of obtaining multiwave interference in the resonance method of two independently oscillating fields has previously been experimentally proved (Rayleigh method) [5]. The splitting of a neutron wave in the system consisting of N resonance coils was also demonstrated, and a neutron spin in each coil flipped with the probability ρ smaller than unity. 2^N neutron waves are formed at the yield of this system that interfere with each other [6, 7].

The use of NRSE with ρ smaller than unity allows one to split the incident neutron wave into four coherent waves in the first device arm placed in front of the sample. The phases of the four neutron waves gathered in the spin echo device arm can be rewritten by the distances between wave fronts (R_{1-2}, R_{2-3}, R_{3-4}, R_{1-4}, R_{1-3}, R_{2-4}) or as time intervals (T_{1-2}, T_{2-3}, T_{3-4}, T_{1-4}, T_{1-3}, T_{2-4}). A neutron split into four waves penetrates into the sample and can be simultaneously scattered from four different points r_1, r_2, r_3, and r_4 or from the common point of space but at different instants of time t_1, t_2, t_3, and t_4. After the scattering act, the second spin echo arm analogous to the first one compensates for the action of the first arm and all four waves reduce to one point of the phase space when leaving a device and interfere with each other. If a neutron interacted with the sample, it reflects on the phases of four interfering waves and, consequently, on the resulting interference curve, as in the case of the standard spin echo. However, when four coherent waves interact simultaneously in four different points of the sample, the signal recorded by a detector carries specific information on high-order correlation functions.

In the given work, the results of neutron experiments on the multiwave resonance spin echo obtained recently at a VVR-M reactor (Petersburg Institute of Nuclear Physics (PINR), Russian Academy of Sciences, Gatchina) are presented. The principle possibilities of observing multiwave interference are shown, and the simple theoretical calculations describing this phenomenon are presented.

THEORY

It is well known that an energy or neutron pulse is coded to the Larmor precession phase in classical NSE devices when a neutron passes through magnetic field [1],

$$\phi = \frac{\gamma_L l_x B}{\upsilon_x}, \tag{1}$$

where ϕ is the neutron precession phase in the magnetic field B with the Larmor precession l_x, γ_L is the gyromagnetic ratio of a neutron, and υ_{Bx} is the velocity projection of a neutron on the normal to the plane-parallel magnetic field boundaries. The method is effective when the change of the velocity component $\delta\upsilon_{Bx} = \upsilon_{Bx} - \upsilon'_{Bx}$ measured in the experiment is smaller than the neutron beam dispersion $D[\upsilon_{Bx}]$. Then spin echo polarization component P_x can be written in the form

$$P_x = P_0 \int f(\upsilon)(\cos(\phi_{\mathrm{I}}(\upsilon_{Bx}) - \phi_{\mathrm{II}}(\upsilon'_{Bx})))d\upsilon, \tag{2}$$

where P_0 is the initial polarization of a neutron beam; $f(\upsilon_{Bx})$ is the velocity distribution function of neutrons; and $\phi_{\mathrm{I}}(\upsilon_{Bx})$ and $\phi_{\mathrm{II}}(\upsilon'_{Bx})$ are the precession phases in the first and second arm, respectively.

Polarization P_x in the balance point $(\phi_{\mathrm{I}}(\upsilon_{Bx}) = \phi_{\mathrm{II}}(\upsilon'_{Bx}))$ as a function of the precession phase in one of the arms $P_x(\phi_{\mathrm{I;II}})$ contains the Fourier transform of distributions both of energies and of pulses transmitted to a neutron. When the transmitted energy is of interest, the precession phase is written in the form of the so-called spin echo time [1],

$$\tau = \frac{\hbar\phi_T}{2E}. \tag{3}$$

When the transmitted pulses are studied, it is written in the form of spin echo length [9]

$$z = \frac{\hbar\phi_T\upsilon\tan(\theta)}{4E}, \tag{4}$$

where $E = m_n\upsilon^2/2$ is the neutron energy and θ is the angle between the normal to the moving direction of a neutron and the magnet field boundaries.

It is convenient to describe the phase ϕ in the NRSE experiments by the quantum-mechanical consideration [8]. A plane neutron wave moving along the x axis with the momentum $\boldsymbol{k} = (k_0, 0, 0)$ can be presented in the form of $\exp(ik_0x)\exp(-i\omega t)$ with the initial wave number k_0 and energy $\hbar\omega$. This wave undergoes Zeeman splitting in magnetic field and, as a result, represents two plane waves with the wave numbers k_+ and k_- as soon as a neutron enters into the field B [4]. Since the total neutron energy $\hbar\omega$ does not change on entering into magnetic field by virtue of the law of conservation of energy conservation, these waves should satisfy the relation

$$\frac{\hbar^2 k_0^2}{2m_n} = \frac{\hbar^2 k_\pm^2}{2m_n} \mp \mu_n B. \tag{5}$$

It is possible to determine k_+; k_-, and the difference Δk between them from this relation,

$$k_\pm = k_0 \pm \frac{\mu_n B}{\hbar\upsilon}, \quad \Delta k = 2\frac{\mu_n B}{\hbar\upsilon}, \tag{6}$$

where m_n, μ_n, and υ are the neutron mass, magnetic moment, and velocity, respectively.

The phases $\phi_+ = k_+x$ and $\phi_- = k_-x$ of both plane waves are not equal to each other on leaving magnetic field B, but in this point the phase difference no longer increases and remains constant $\phi = (k_+ - k_-)l = (2\mu_n/\hbar\upsilon_n)Bl$, where l is the path passed by a neutron in a uniform magnetic field. When a magnetic field is nonuniform along the x axis, the expression for the phase takes the form of the integral along the trajectory $\phi_l = \int(k_+(x) - k_-(x))dx$. Thus, from the quantum point of view, the Larmor precession is the integral of the difference of two wave numbers split by the magnetic field of plane waves.

In the NRSE method, the neutron spin precession occurs in the nonstationary field of the system consisting of two resonance coils. A neutron interacts with the radiofrequency field by means of transfer of a virtual photon with the $\hbar\omega$ energy. The field frequency in this method is determined so that the virtual photon energy is identical to the Zeeman energy difference between two spin states of a neutron in static field,

$$\hbar\omega_{\mathrm{RF}} = -2\mu_n B. \tag{7}$$

Thus, this expression is the resonance condition.

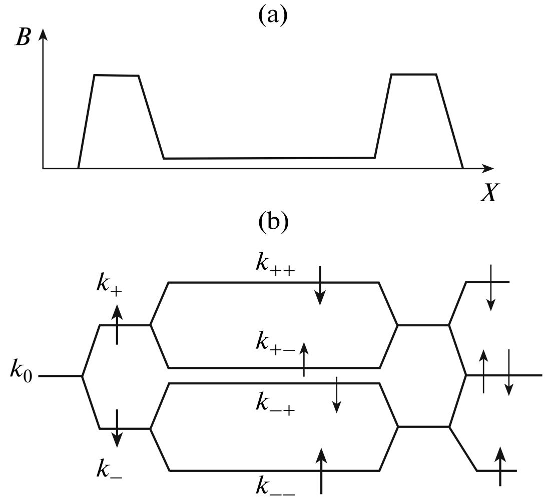

Fig. 1. Sketch of the magnetic field system consisting of two radiofrequency coils (the static field B_0, the coil length l, and the distance L between them) with the guiding field B_1 between them (a). (k, x) Is the diagram of the dependence of neutron wave vectors on the position along beam (b).

The probability that a neutron will change its spin state depends on the radiofrequency field strength $B_{\rm RF}$ and has the form

$$\rho = \sin^2(b_\tau/2), \quad b = -\frac{2\mu_n}{\hbar}B_{\rm RF}, \quad \tau = l/\upsilon, \quad (8)$$

here, l is the flipper length and υ is the neutron velocity.

It follows from (8) that the expression $b\tau = \pi$ is the condition for the full spin flip.

This means that the radiofrequency field amplitude, the radiofrequency coil length, and the neutron velocity must conform so that the polarization vector turns through π as a neutron passes through it.

As was noted in the Introduction, the repeated splitting of a neutron wave results if several RCs are used with the spin flip probability ρ smaller than unity. In this case a number of spin states of one neutron inside each coil doubles; therefore, the number of neutron waves at the coil exit doubles as well. Figure 2 helps to analyze the phase shift between two pairs of neutron waves formed after propagation of the system consisting of two RCs. When the radiofrequency field is switched off, this phase shift is proportional to hatched region I and equal to

$$\phi_1 = \int(k_{+-}(x) - k_{-+}(x))dx, \quad (9)$$

where the integral is taken with respect to the length of the system consisting of two flippers and k_{+-} and k_{-+} denote the trajectories of the neutron wave splitting in the first flipper. This is known as the DC precession, i.e., the Larmor precession only in a static field of flippers. When the radiofrequency field is switch on (spin flip probability is $\rho = 1$), the precession value in a zero,

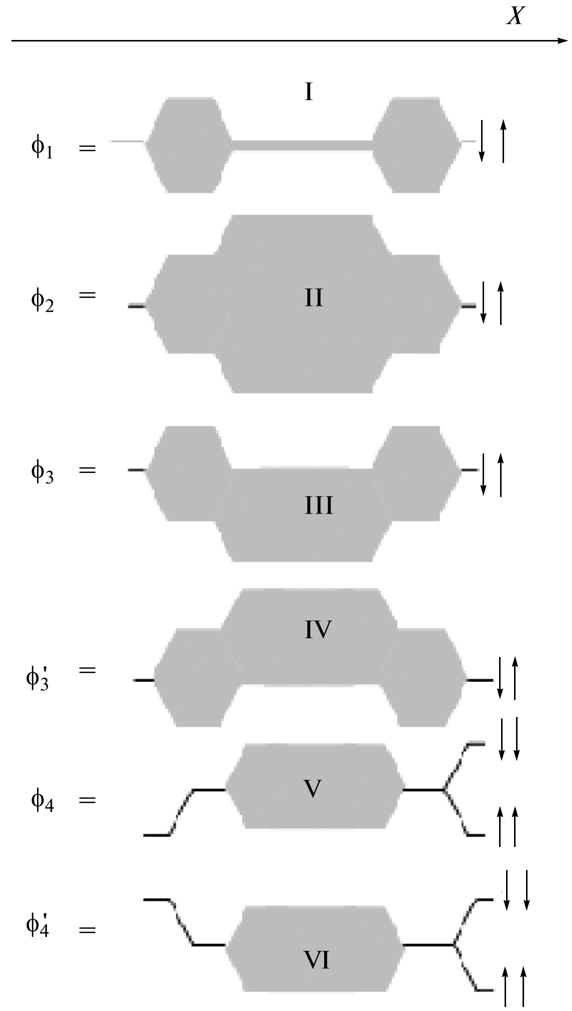

Fig. 2. Scheme of the neutron wave splitting in the k space: x is the neutron wave motion direction; envelope lines are the states of the k vector. Dashed regions show the phase shifts between neutron wave pairs after propagation of the system consisting of two RCs.

i.e., in a weak, field (so-called ZF precession) is proportional to region II and equal to

$$\phi_2 = \int(k_{++}(x) - k_{--}(x))dx. \quad (10)$$

As was noted, when $0 < \rho < 1$, all four k levels in the space between flippers will be occupied by a neutron. Shaded regions III and IV in Fig. 2 correspond to the phase shifts

$$\phi_3 = \int(k_{++}(x) - k_{-+}(x))dx, \quad (11)$$

$$\phi_3' = \int(k_{+-}(x) - k_{--}(x))dx. \quad (12)$$

One can show that $\phi_3 = \phi_3' = (\phi_1 + \phi_2)/2$, so that the phase of the interference terms corresponds to

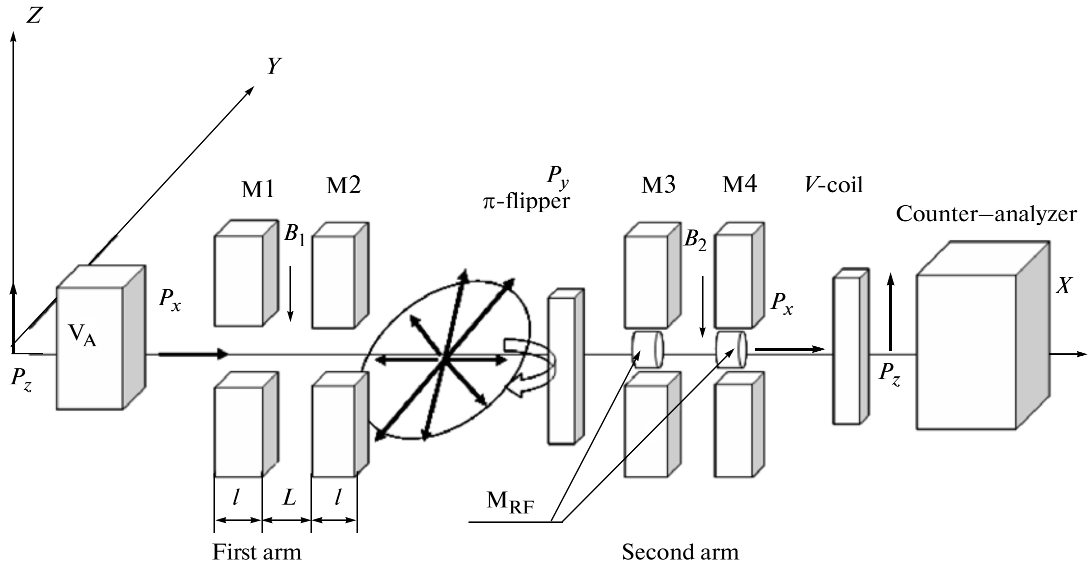

Fig. 3. Schematic image of a spin echo device with the resonance second arm (two radiofrequency resonance coils). Field extent B_0 inside M1–M4 magnets is $l = 0.2$ m; the distance between magnets is $L = 0.6$ m. The coils with an RF field directed along the beam are located inside the second arm magnets.

one-half the total I and II area. The interference terms with the phases

$$\phi_4 = \int (k_{++}(x) - k_{+-}(x))dx, \quad (13)$$

$$\phi'_4 = \int (k_{-+}(x) - k_{--}(x))dx \quad (14)$$

correspond to shaded regions V and VI. They are also equal to each other and to one-half the II and I area difference, $\phi_4 = \phi'_4 = (\phi_2 + \phi_1)/2$.

In our experiment network, the first device arm operated in the classical spin echo mode, whereas the second arm represented the resonance arm in which the spin flip was realized with the different probability ρ.

Denote ϕ_{11} by the ϕ_1 phase accumulated in the first arm and ϕ_{21}, ϕ_{22}, and ϕ_{23} by the ϕ_1, ϕ_2, and ϕ_3 phases accumulated in the second arm, respectively. The ϕ_{11} phase of the first arm is equivalent to the ϕ_1 phase in (9). The ϕ_{21}, ϕ_{22}, and ϕ_{23} phases are equivalent to the ϕ_1, ϕ_2, and ϕ_3 phases in (9), (10), and (11).

When the change of neutron velocity is negligible and the velocity distribution changes insignificantly, $f(\upsilon) \approx f(\upsilon')$, where υ and υ' are the neutron velocity before and after the scattering act, the following expression for the polarization component P_x is valid:

$$\begin{aligned} P_x = P_0 \int f(\upsilon)(A_C\cos(\phi_{11}(\upsilon) - \phi_{21}(\upsilon')) \\ + A_I\cos(\phi_{11}(\upsilon) - \phi_{23}(\upsilon')) \\ + A_R\cos(\phi_{11}(\upsilon) - \phi_{22}(\upsilon')))d\upsilon. \end{aligned} \quad (15)$$

Here A_C, A_R, and A_I are the amplitudes of classical, resonance, and interference spin echo signals that, according to [5], can be presented in the form

$$\begin{aligned} A_C &= (1-\rho)^2, \\ A_I &= 2(1-\rho)\rho, \\ A_R &= \rho^2. \end{aligned} \quad (16)$$

Note that in our case $f(\upsilon) \equiv f(\upsilon')$, since scattering from the sample is absent.

In the experiments we have observed interference patterns corresponding to the ϕ_1, ϕ_2, and ϕ_3 phases with the A_C, A_R, and A_I amplitudes (16), since only the P_x polarization component was measured, whereas the interference pattern corresponding to ϕ_4 has been not studied, since it is detected for the P_z polarization component [5].

Each of these neutron waves interacts with a field according to an inherent spin state and composes an inherent phase. Neutron waves interfere after propagation through this system. This system of resonators and magnetic fields allows us to manipulate easily the neutron state—for example, splitting it into a set of waves, the phases and amplitudes of which are controlled by the experimentalist.

EXPERIMENTAL

The experiments were carried out at a spin echo device arranged at the 14th VVR-M reactor beam in Gatchina. The device network is shown in Fig. 3. Neu-

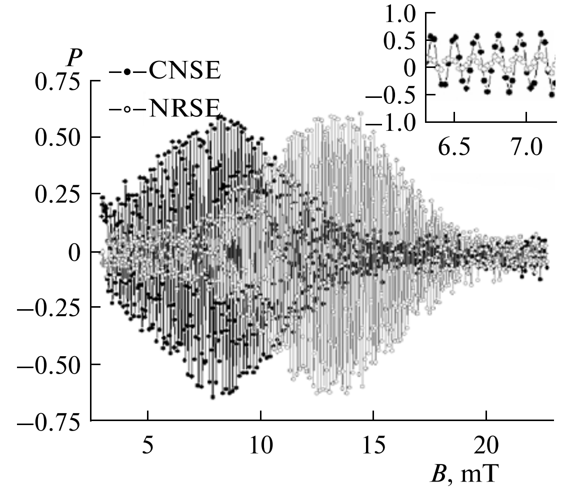

Fig. 4. Spin echo signal obtained for switched-off and -on RF coils as a field function of M1 and M2 magnets ($B = B_{M1} = B_{M2}$) for $B_{M3} = B_{M4} = 5.3$ mT. The field amplitude is $B_{RF} = 0.7$ mT, which corresponds to $\rho = 1$. The RF field resonance frequency of the second arm coils is 200 kHz. The fragment of signal is shown in the insert.

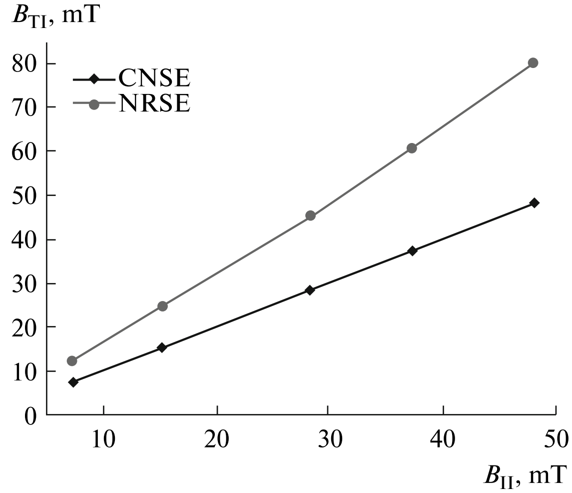

Fig. 5. Dependence of the position of balance point B_{TI} on the fields of the second arm magnets ($B_{II} = B_{M3(M4)}$).

trons polarized along the Z axis with the wavelength $\lambda = 2$ Å and $\Delta\lambda/\lambda = 0.02$ come to a vector analysis (VA) block where the neutron polarization vector turns through 90°; then neutrons nonadiabatically enter into a magnetic field with the direction Z of the first device arm consisting of M1 and M2 electromagnets. A neutron wave splits into k_- and k_+ in the B_{M1} and B_{M2} fields, as a result of which the phase difference ϕ_I between these waves (9) accumulates at the output of a magnet. Note that the wavelength uncertainty $\Delta\lambda/\lambda = 0.02$ causes the phase uncertainty $\Delta\phi_I/\phi_I = 0.02$, so that for ϕ_I values larger or of order of 100π this uncertainty reaches the value of 2π and a neutron beam becomes completely dephased.

After flipping in a flipper through 180° about the Yaxis a neutron wave comes to the second arm consisting of M3 and M4 magnets. The presence of the π flipper between the first and second arms leads to the fact that the phase difference ϕ_{II} in the second arm compensates the phase difference ϕ_I in the first arm, and, if $\phi_{II} - \phi_I = 0$, then the beam phasing at the output of the B_{M2} is completely recovered. According to (7), radiofrequency coils tuned to resonance with the field of M3 and M4 magnets are arranged inside M3 and M4 magnets. The radiofrequency field amplitude defines the spin flip probability ρ (8), which, in turn, defines the neutron wave amplitudes (16). The phase shifts of the waves ϕ_1, ϕ_2, and ϕ_3 are determined from Eqs. (9)–(11), respectively. Then a neutron beam comes into a $\pi/2$ coil, where the neutron polarization vector is turned through 90°. After that the neutrons with the spins polarized along the Z direction are detected and the neutrons with the spins opposite to a field are absorbed by an analyzer.

The scanning in the experiments was carried out along the field of M1 and M2 magnets for the fixed fields in M3 and M4 magnets and the MRF. Three measurement modes were used.

I. Classical spin echo (radiofrequency field was switched off).

II. Resonance spin echo. In this mode the resonance radiofrequency field amplitude was selected so that the full spin flip $\rho = 1$ was realized for the neutrons with the wavelength $\lambda = 2.0$ Å. The device has the classical first arm (M1 and M2 magnets) and the resonance second arm (M3 and M4 magnets).

III. Multiwave spin echo. The radiofrequency field amplitude in M3 and M4 magnets was tuned so that a full spin flip was not realized ($0 < \rho < 1$). Parts of the neutron wave with different phases interfere at the output of this system, as a result of which new interference components of the spin echo signal appear.

RESULTS

The dependences of the neutron beam polarization on the magnet field $B_{M1} = B_{M2}$ for the classical (CNSE) and resonance (RNSE) spin echo mode are shown in Fig. 4. The field in the magnets of the second arm is $B_{M3} = B_{M4} = 7.3$ mT. It is seen from the right upper fragment that the signal has a sinusoidal modulation. Both dependences have a characteristic form of a neutron spin echo signal (a modulated sinusoidal signal the amplitude of which symmetrically decreases when leaving the certain field B_C). The signal amplitudes are approximately identical, whereas the positions of signal centers substantially differ.

The position of the spin echo signal maximum denotes a balance point ($\phi_{II} - \phi_I = 0$). The dependence of B_{TI} on the field $B_{II} = B_{M3(M4)}$ is shown in Fig. 5. It is seen that, in the NRSE mode, the dependence $B_{TI}(B_{II})$

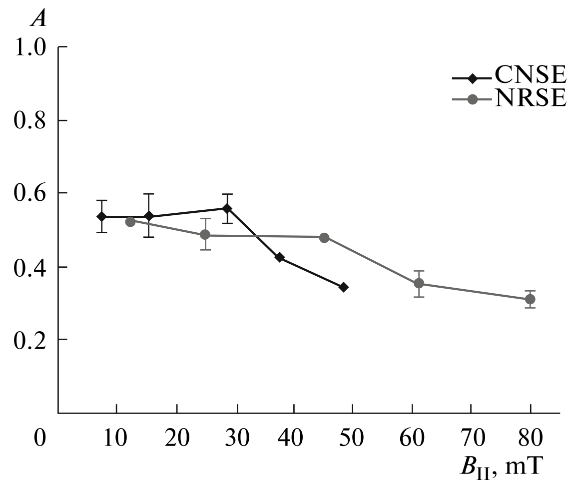

Fig. 6. Spin echo signal amplitude as a function of the first arm field B_I for the classical (CNSE) and resonance (NRSE) spin echo modes.

is steeper than in the NSR mode. This means that the phase ϕ_{II} is larger than that in the NRSE mode for identical values of B_{II}. Since the phase ϕ defines the quantity of the scanning parameter of the spin echo time τ or the spin echo length z, the measurement range in the NRSE mode will be larger than in the classical NSE. The spin echo signal amplitude depending on the position of the balance point is presented in Fig. 6 by the $B_{TI} = B_{M1(M2)}$ coordinates. Rather small value of polarization $P = 0.6$ near $B_I = 0$ is related to the deterioration in adiabaticity conditions of polarization propagation for the small fields B_{M1}–B_{M4}, whereas a decrease in the amplitude with an increase in B_I is caused by the field nonuniformity along the beam cross section, which decreases the spin echo device sensitivity.

The magnet field dependence of the neutron polarization $B_{M1}(B_{M2})$ for the interference spin echo mode is shown in Fig. 7. The measurements were carried out in the fields $B_{M3} = B_{M4} = 35$ mT; the resonance frequency of radiofrequency field was $f = 1$ MHz for the probability of spin flip $\rho = 0.42$. The experimental curve consists of three spin echo signals imposed on each other the centers of which are shifted relative to each other. The central signal has the largest amplitude. Two signals with smaller amplitudes are located to the left and right of the central signal. The positions of the signal maxima to the left and right correspond to the positions of the maxima of classical and resonance spin echo signals obtained in the same fields $B_{M3} = B_{M4} = 35$ mT. However, the central signal is a new element. This signal is caused by the interference of neutron waves involved in the formation of classical and resonance spin echo.

The appearance of an interference part of the spin echo signal is also observed in Fig. 8. The curves presented in the figure are the classical and interference spin echo measured in the same field B_{M3} and B_{M4} as the curves in Fig. 5, but at $\rho = 0$ (CNSE) and $\rho = 0.74$ (interference (NRSE)). In the interference mode, the amplitude of the right spin echo signal almost coincides with the amplitude of the classical spin echo signal. It is similar to the situation in Fig. 4, in which the resonance and classical spin echo signals have identical amplitudes. Thus, the graph shows the low sensitivity of the amplitude of the resonance signal part to a decrease in ρ from 1 to 0.74. However the appearance of the notable interference part indicates that ρ is smaller than unity.

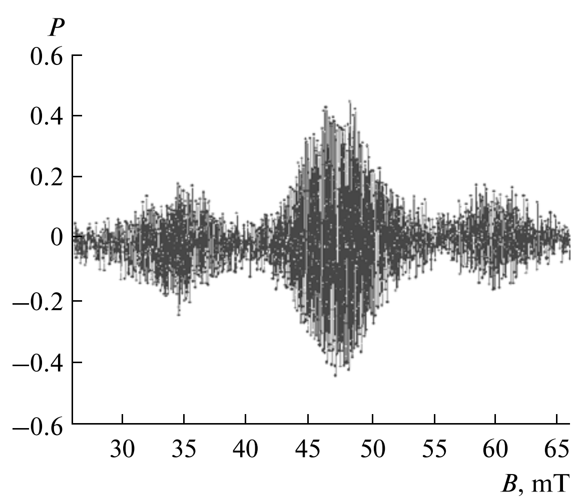

Fig. 7. Spin echo signal as a function of the field of M1 and M2 magnets in the fields $B_{M3} = B_{M4} = 35$ mT. The signal was obtained for switched-on RF coils with the field amplitude $B_{RF} = 0.35$ mT corresponding to $\rho = 0.42$. The resonance frequency of second arm flippers is 1 MHz.

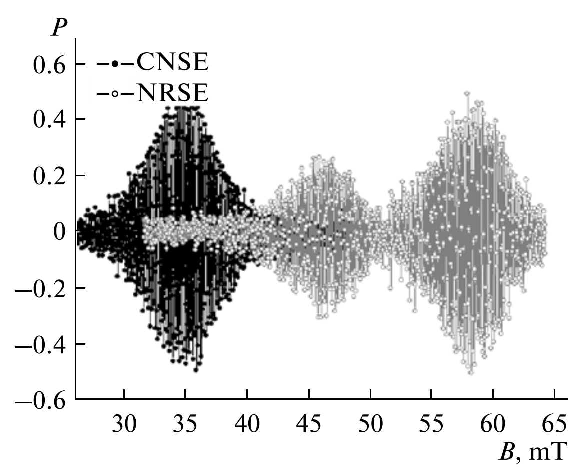

Fig. 8. Spin echo signal as a function of the field of M1 and M2 magnets in the fields $B_{M3} = B_{M4} = 35$ mT. The signal was obtained for switched-off and -on RF coils with the field amplitude $B_{RF} = 0.7$ mT corresponding to $\rho = 0.74$. The resonance frequency of the second arm flippers is 1 MHz.

CONCLUSIONS

The performed investigations are both of fundamental and of practical interest. The potential possibility to measure the high-order correlation functions is an attractive prospect from the viewpoint of experimental physics. For NRSE with $\rho < 1$, the neutron wave splitting into four coherent parts means the possibility to observe scattering simultaneously in four different points r_1, r_2, r_3, and r_4 or in the same space point but at different instants of time t_1, t_2, t_3, and t_4. Our experiments showed the principal possibility to realize the four-wave neutron resonance spin echo.

The performed experiment also is the first demonstration of the possibility to realize a resonance spin echo device based on a VVR-M reactor (Gatchina). It has been shown that the measurement NRSE range for the given device configuration is larger than the classical NSE. The spin flip probability in the resonance spin echo device arm results in the appearance of an additional interference contribution to the spin echo signal. This contribution is well observed in the experimental curve and can be used for the spin echo device tuning.

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