Neutron polarization analysis:
from diagonal to spherical 3D

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Jane Brown (ILL Grenoble)

E. Lelievre-Berna (ILL Grenoble)

L.-P. Regnault (CEA Grenoble)

Literature

- **Fundamental equations**

  S.V. Maleyev, Zh. Exp. Theor. Phiz. 40 (1961) 1224,

  JETP 13 (1961) 860

  M. Blume, Phys. Rev. 130 (1963) 1670

- **Longitudinal (diagonal) polarization analysis**

  R.M. Moon, T.Riste, W.C. Koehler, Phys. Rev. 181 (1969) 920

- **Spherical polarization analysis**

  P.J. Brown, Physica B 297 (2001) 198

Neutron polarization analysis ...

1. Fundamental equations

2. Longitudinal (diagonal) polarization analysis

1. Spherical polarization analysis

Fundamental equations

Fermi's golden rule:

$$\frac{d^2\sigma^{ss'}}{d\Omega' dE'} = \sum_q P_q \sum_{q'} \frac{k'}{k}\left(\frac{m_0}{2\pi\hbar^2}\right)^2$$

$$\times \left| \left\langle s'q' \middle| \int d\mathbf{r}\, e^{i\mathbf{K}\cdot\mathbf{r}} V(\mathbf{r}) \middle| sq \right\rangle \right|^2 \delta\left(\frac{\hbar^2}{2m_0}(k'^2 - k^2) + E_{q'} - E_q\right)$$

s, s'	neutron spin states
q, q'	target states
V(**r**)	interaction potential

NEUTRONS FOR SCIENCE

Fundamental equations

Scattering cross-section:

$$\frac{d^2\sigma^{ss'}}{d\Omega\, dE} = \sum_q P_q \sum_{q'} \frac{k'}{k} \left| \langle q' | \sum_i U_i^{ss'} \exp\left(i\vec{Q}\vec{r}_i\right) | q \rangle \right|^2 \times \delta(E - E')$$

Scattering amplitudes:

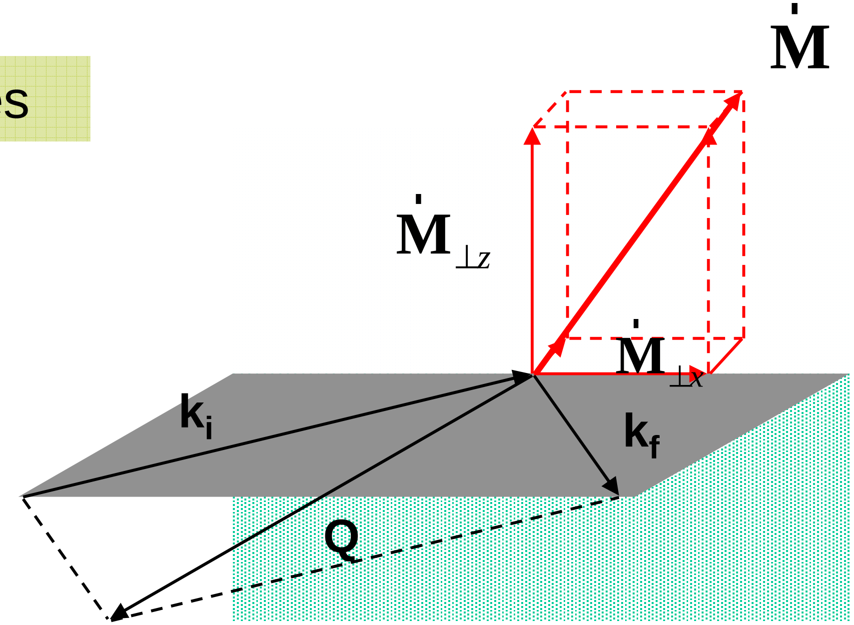
$$U_i^{ss'} = \langle s' | \left(b_i - \vec{M}_{\perp i} \cdot \vec{\sigma} + B_i \vec{I}_i \cdot \vec{\sigma} \right) | s \rangle$$

nuclear — magnetic — nuclear spin

Magnetic scattering

- only projection of **M⊥Q** contributes

$$\vec{M}_\perp(\vec{Q}) = \vec{e}_Q \times \vec{M}(\vec{Q}) \times \vec{e}_Q$$



$$\vec{M}(\vec{Q}) = \sum_j \vec{M}_j(Q)\exp\left(i\vec{Q}\vec{r}_j\right)\exp\left(-W_j\right)$$

- magnetic scattering amplitude

$$M_j(Q) = \frac{\gamma e^2}{2mc^2}\, gS_j\ f(Q)$$

Neutron polarization analysis ...

1. **Fundamental equations**
   - *only* $\mathbf{M} \perp \mathbf{Q}$ *gives rise to scattering*
   - *beam polarization depends on* $(\sigma . \mathbf{M}_\perp)$

1. **Longitudinal (diagonal) polarization analysis**

2. **Spherical polarization analysis**

Simplified scattering amplitudes

Presence of a guiding field **H || z:**

$$U^{++} = b - M_{\perp z} + B\vec{I}_z$$

$$U^{--} = b + M_{\perp z} - B\vec{I}_z$$

$$U^{+-} = -\left(M_{\perp x} + iM_{\perp y}\right) + B\left(\vec{I}_x + i\vec{I}_y\right)$$

$$U^{-+} = -\left(M_{\perp x} - iM_{\perp y}\right) + B\left(\vec{I}_x - i\vec{I}_y\right)$$

"Moon's golden rule":

$\vec{M}_\perp \parallel \vec{\sigma}_n$	non spin-flip (NSF)	U^{++}, U^{--}
$\vec{M}_\perp \perp \vec{\sigma}_n$	spin-flip (SF)	U^{+-}, U^{-+}

H = 0: total scattering

- absence of quantization axis

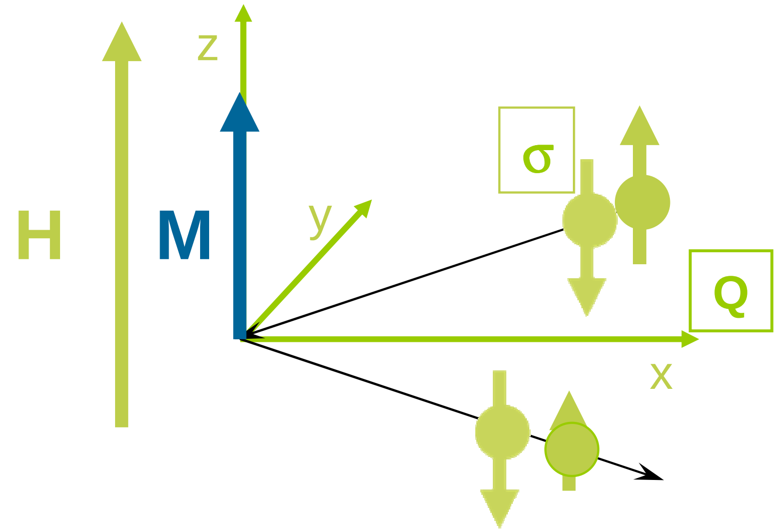
$$\langle \mathbf{U}\mathbf{U}^{+} \rangle \rightarrow b^2 + \left| \langle M_{\perp} \rangle \right|^2$$

- nuclear & magnetic scattering are mutually independent
- total scattering is given by sum of intensities

$$I_{tot} \approx \left| F_N \right|^2 + \left| F_M \right|^2$$

H ⊥ Q: ferromagnets

- magnetically saturated (single domain) ferromagnetic crystal
- applied magnetic field ≈ 1 T



$$I^{+} \approx \left(F_N + F_M\right)^2$$

$$I^{-} \approx \left(F_N - F_M\right)^2$$

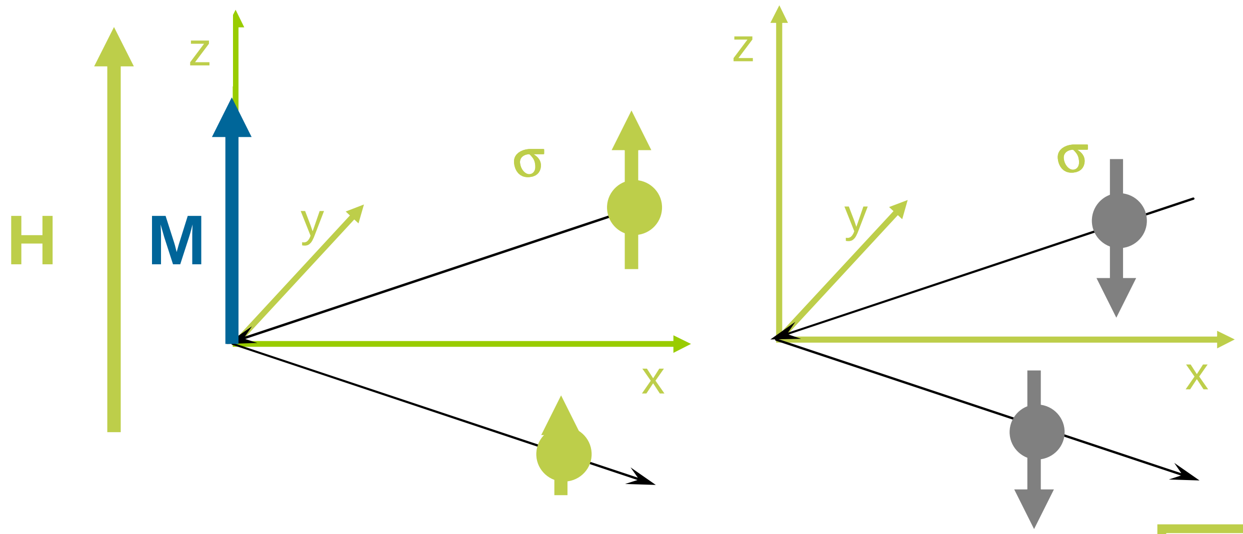
- beam polarization

$$P = \frac{I^{+} - I^{-}}{I^{+} + I^{-}}$$

diffracted beam highly polarized for $|F_N| \approx |F_M|$

Magnetic formfactor studies

- magnetically saturated (single domain) ferromagnetic crystal
- applied magnetic field < 10 T
- polarized incident beam



$$I^{+} \approx (F_N + F_M)^2$$

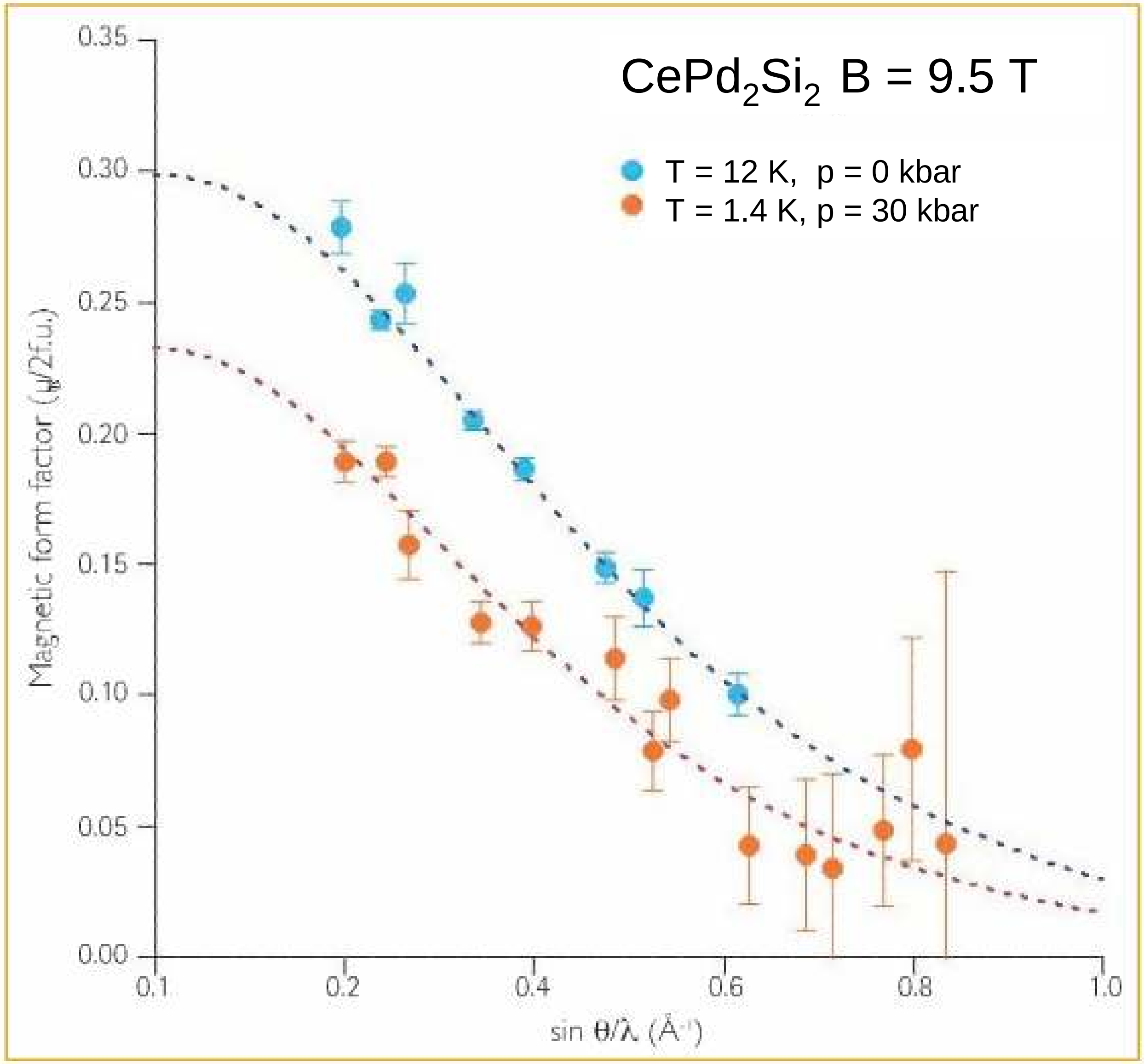
$$I^{-} \approx (F_N - F_M)^2$$

- flipping ratio in diffracted beam

$$R = \frac{I^{+}}{I^{-}} \approx \frac{|F_N + F_M|^2}{|F_N - F_M|^2}$$

- extract $F_M(Q)$ and reconstruct $M(R)$

Formfactor of Ce

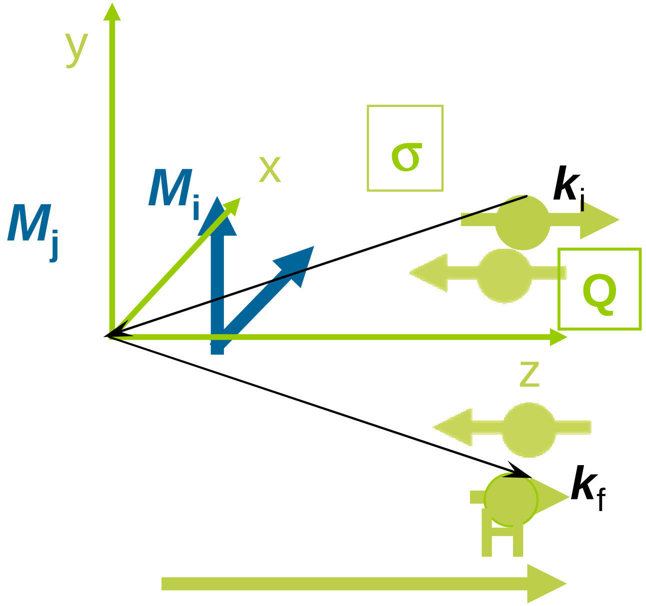


- *4f* electron system (Ce)
- ordering at T ≈ 10 K (p = 0)
- quantum critical point ≈ 28 kbar
- reduction of induced magnetization
- change of form factor shape

Valence change Ce^{3+} to $Ce^{(3+\delta)+}$?

N. Kernavanois, R. Sadykov, E. Resouche, S. Raymond et al., 2004

H || Q: chirality



$$U^{++} = U^{--} = b$$

$$U^{\pm\mp} = -\left(M_{\perp x} \pm i M_{\perp y}\right)$$

$$I^{NSF} \approx b_i b_j^+$$

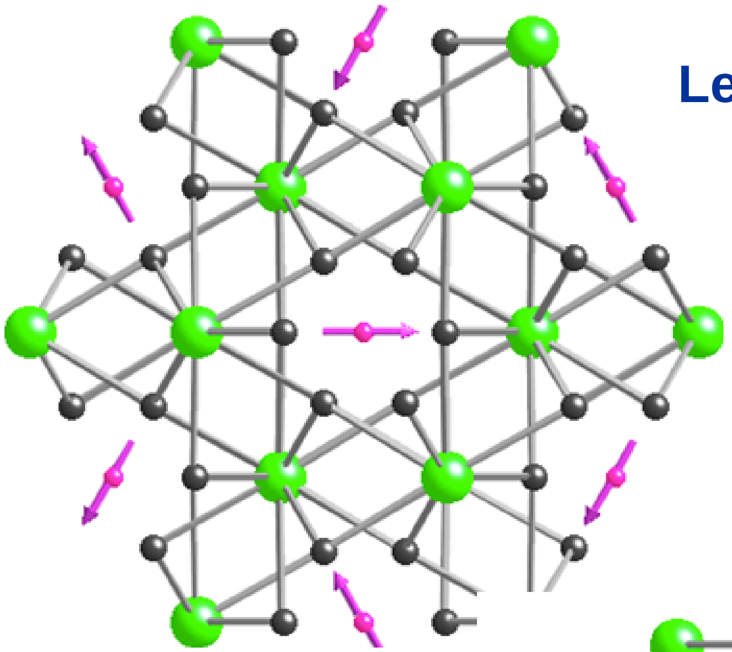
$$I^{SF\pm} \approx \vec{M}_{\perp i} \cdot \vec{M}^+_{\perp j} \mp \mathrm{Im}\left(\vec{M}_{\perp i} \times \vec{M}^+_{\perp j}\right)_\perp$$

- **beam polarization**

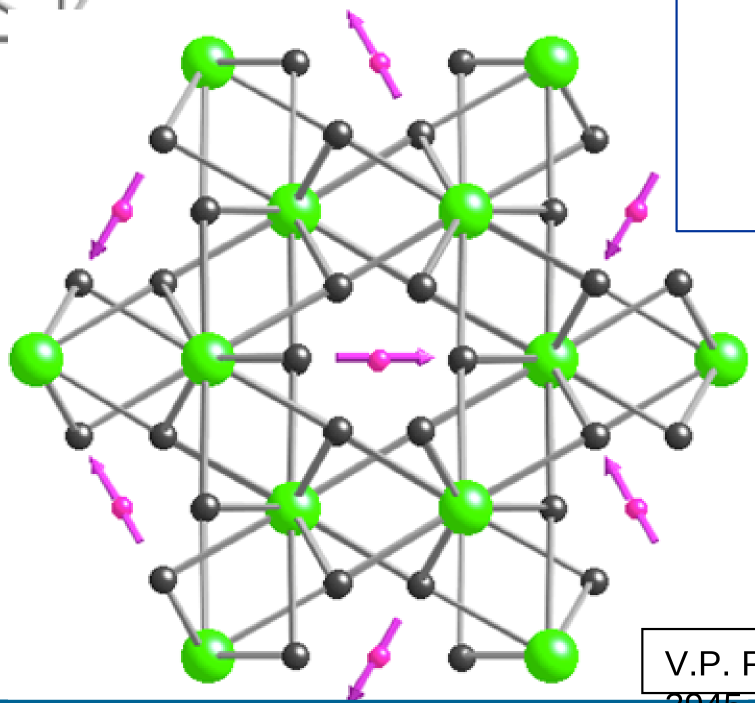
$$P = \frac{I^{SF+} - I^{SF-}}{2I^{NSF} + I^{SF+} + I^{SF-}}$$

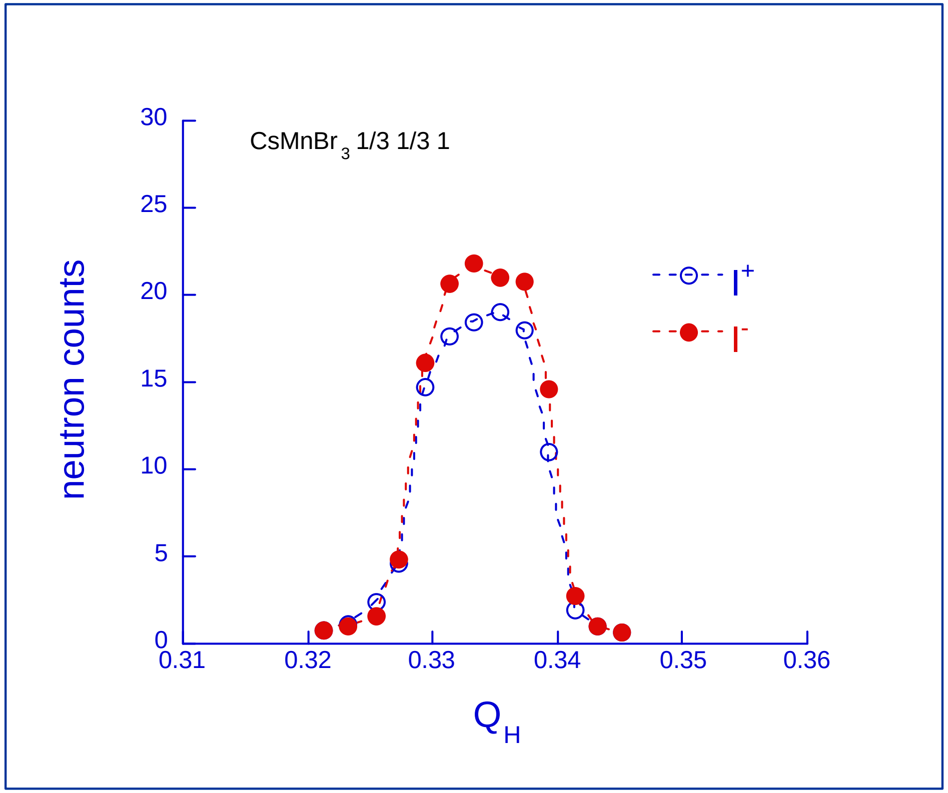
Chiral correlations in $CsMnBr_3$

Left-handed



Right-handed





Polarization in (1/3 1/3 1) rocking curves

V.P. Plakhty, et al., Phys. Rev. Lett. 85 (2000) 3942-3945

Selection rules I

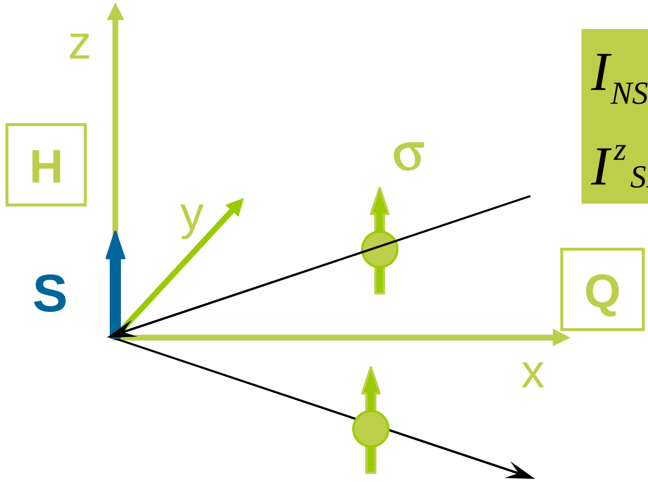
$\mathbf{H \perp Q}$

$$U^{++} = b - M_{\perp z}$$

$$U^{--} = b + M_{\perp z}$$

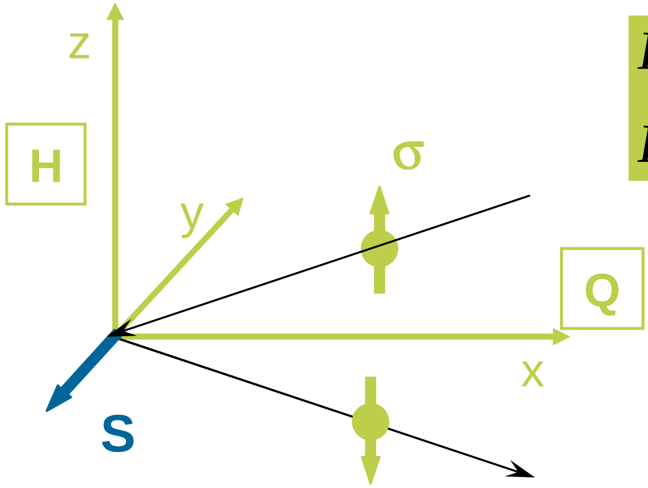
$$U^{+-} = -iM_{\perp y}$$

$$U^{-+} = iM_{\perp y}$$



$$I_{NSF} \approx (b + M_{\perp})^2$$

$$I^z{}_{SF} = 0$$



$$I_{NSF} \approx b^2$$

$$I_{SF} \approx M_{\perp}{}^2$$

Selection rules II

H||Q

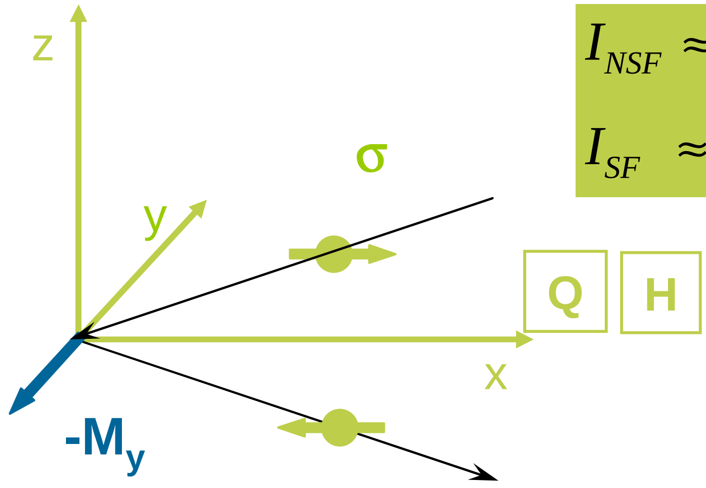
$$U^{++} = b$$

$$U^{--} = b$$

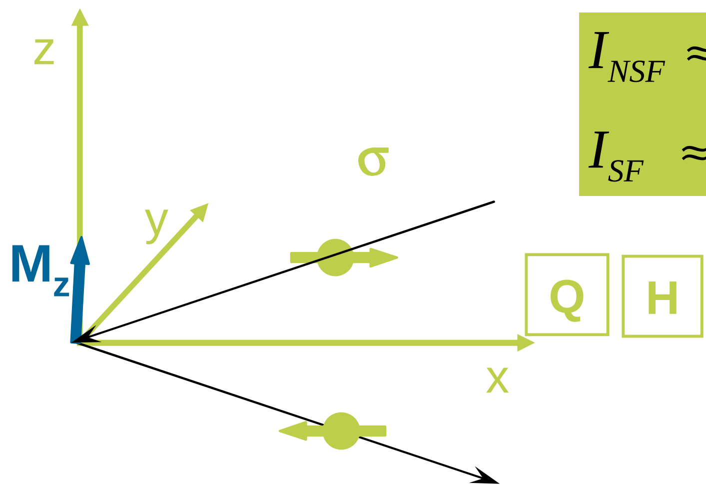
$$U^{+-} = -\left(M_{\perp y} + iM_{\perp z}\right)$$

$$U^{-+} = -\left(M_{\perp y} - iM_{\perp z}\right)$$

- **all magnetic scattering is spin-flip**
- **non spin-flip is nuclear (& incoherent)**
- **chirality produces polarization**



$$I_{NSF} \approx b^2$$
$$I_{SF} \approx \left|M_{\perp y}\right|^2$$



$$I_{NSF} \approx b^2$$
$$I_{SF} \approx \left|M_{\perp z}\right|^2$$

Diagonal polarization analysis

- **Partial intensities (polarized beam):**

$$I_x^{NSF} \approx N^2 + \frac{1}{3} I_{SI} \qquad I_x^{SF} \approx M_{\perp y}^2 + M_{\perp z}^2 + \frac{2}{3} I_{SI}$$

$$I_y^{NSF} \approx \left(N + M_{\perp y}\right)^2 + \frac{1}{3} I_{SI} \qquad I_y^{SF} \approx M_{\perp z}^2 + \frac{2}{3} I_{SI}$$

$$I_z^{NSF} \approx \left(N + M_{\perp z}\right)^2 + \frac{1}{3} I_{SI} \qquad I_z^{SF} \approx M_{\perp y}^2 + \frac{2}{3} I_{SI}$$

- **Use difference signal to extract information:**

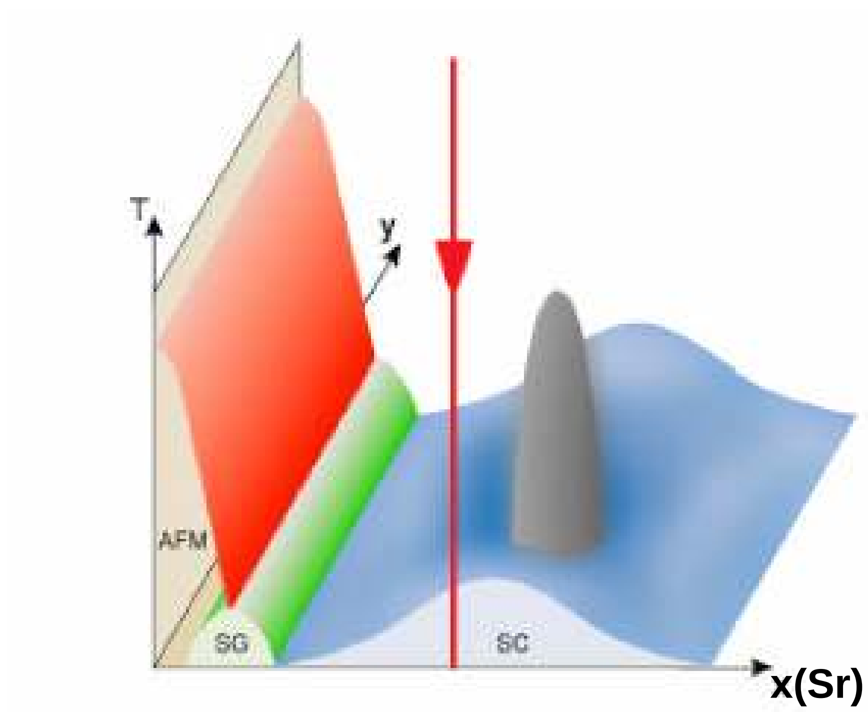
$$\chi_y'' \approx M_{\perp y}^2 \approx I_x^{SF} - I_y^{SF}$$

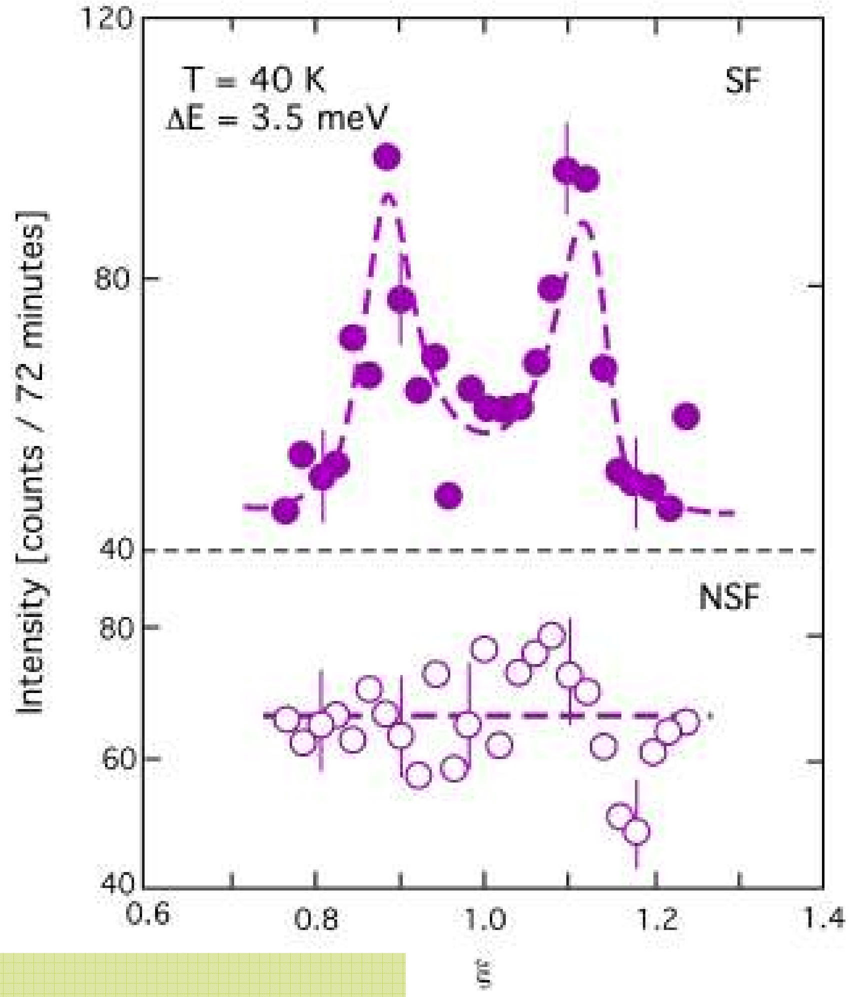
$$\chi_z'' \approx M_{\perp z}^2 \approx I_x^{SF} - I_z^{SF}$$

Helmholtz setup on IN20

N – M separation: simple

Directly use selection rule for I_x SF/NSF





$La_{1.86}Sr_{0.14}CuO_4$: pure spin fluctuations

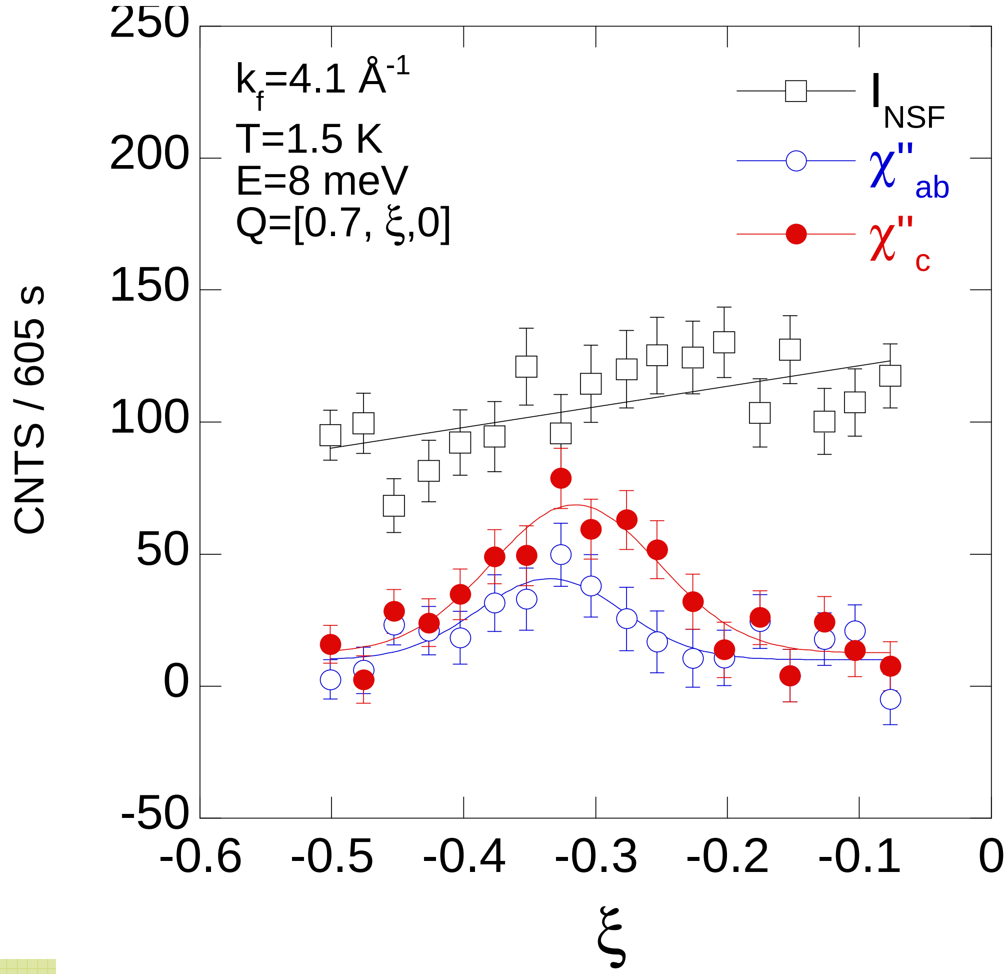
G. Aeppli, T.E. Mason, S.M. Hayden. H.A. Mook, J. Kulda, Science 278 (1997) 1432

N – M separation: elaborate

- *difference signal:*

$$\chi_{y,z}'' \approx M_{\perp y,z}^2 \approx I_x^{SF} - I_{y,z}^{SF}$$

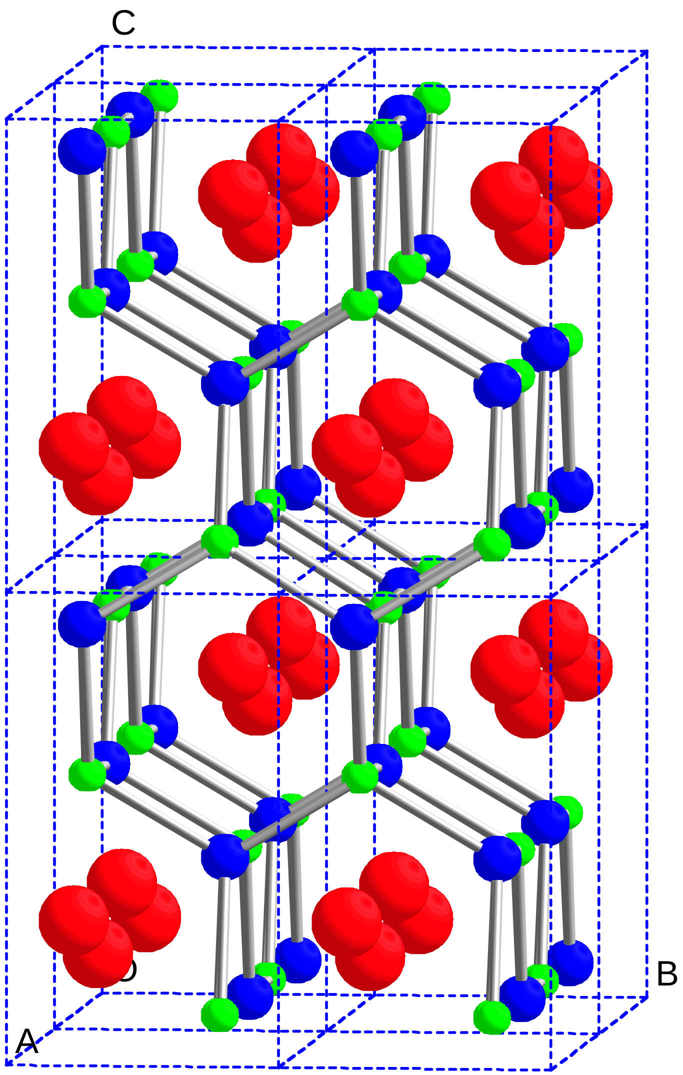
- *no background in difference signal!*
- *works equally well for a flat response!*



Sr_2RuO_4 (S = 1 pairing):
polarization anisotropy

M. Braden et al., Phys. Rev. Lett. 92 (2004) 097402

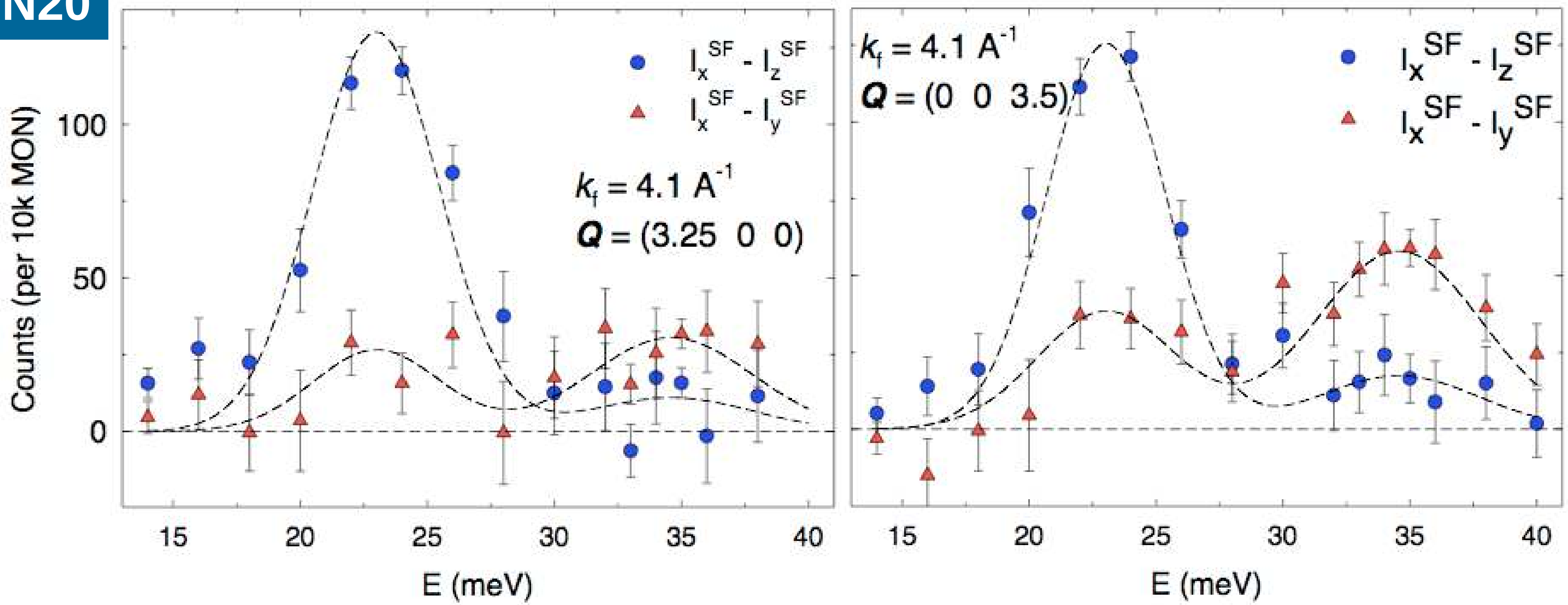
CePtSn



- structure TiNiSi-type (*Pnma*)
- strong Kondo effect (T_K = 10 K)
- antiferromagnet (T_N = 7.5 K)
- RKKY exchange interaction
- propagation ***q*** = (0 0.5 0) + spin slips
- first order (?) transition T_M = 5 K
- magnetization anisotropy (***a*** easy)

Crystal field in CePtSn

IN20



$$I_x^{SF} - I_{y,z}^{SF} \propto \frac{k_f}{k_i} F^2(Q)\exp(-2W) \times \sum_{mm'} n_m |\langle m'|J_{y,z}|m\rangle|^2 \delta(\omega_{m'} - \omega_m - \omega)$$

Crystal field in CePtSn

Diagonal matrix elements

$$\left|\langle n|J_i|n\rangle\right|^2$$

supplied by bulk measurements

	Experiment	Theoretical calculations	
		Monoclinic H_{CF}	Orthorhombic H_{CF}
	(this work)	(this work)	(Diviš *et al.*[a])
Δ_1(meV)	23.0(4)	24.0(8)	23.3
Δ_2(meV)	34.6(7)	34.6(–)	35.2
I_1/I_2	1.45(9)	1.34(25)	0.758
$\|\langle 0\|J_a\|1\rangle\|^2$	262(71)	270(15)	324
$\|\langle 0\|J_b\|1\rangle\|^2$	777(54)	715(12)	385
$\|\langle 0\|J_c\|1\rangle\|^2$	166(40)	167(12)	168
$\|\langle 0\|J_a\|2\rangle\|^2$	474(69)	518(20)	20
$\|\langle 0\|J_b\|2\rangle\|^2$	113(56)	101(12)	405
$\|\langle 0\|J_c\|2\rangle\|^2$	245(32)	259(16)	730

Complete CF wave functions

$$|\Psi_0\rangle = (0.62+0.42i)|+1/2\rangle + (0.15+0.43i)|-3/2\rangle - 0.48|+5/2\rangle$$

$$|\Psi_1\rangle = (-0.56+0.13i)|+1/2\rangle + (0.75+0.06i)|-3/2\rangle - 0.32|+5/2\rangle$$

$$|\Psi_2\rangle = (-0.30+0.15i)|+1/2\rangle - (0.28-0.39i)|-3/2\rangle + 0.82|+5/2\rangle$$

B. Janousova et al., Phys. Rev. B69 (2004) 220412

Neutron polarization analysis ...

1. **Fundamental equations**
   - *only* $\mathbf{M}_\perp\mathbf{Q}$ *gives rise to scattering*
   - *beam polarization depends on* $(\boldsymbol{\sigma} . \mathbf{M}_\perp)$

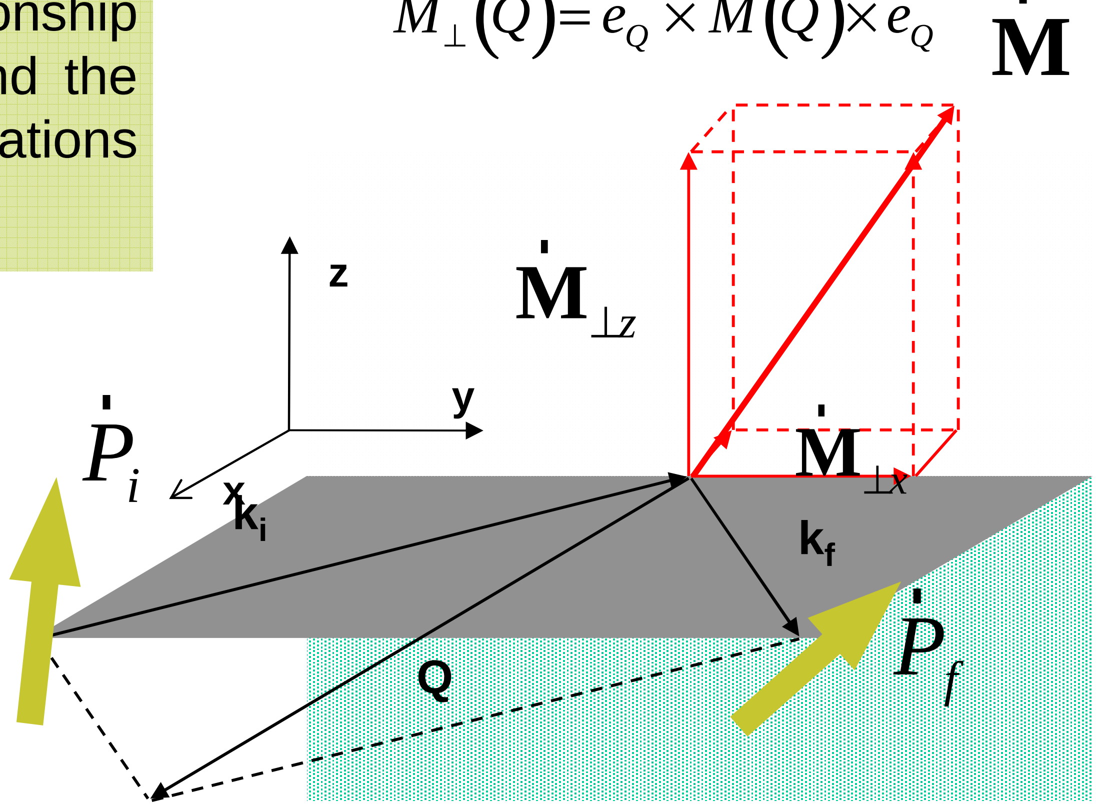
1. **Longitudinal (diagonal) polarization analysis**
   - *beam polarization by net magnetization* $\mathbf{M}_\perp$ (linear or chiral)
   - *spin-flip* $(\boldsymbol{\sigma} \perp \mathbf{M}_\perp)$ *and non-spin-flip scattering* $(\boldsymbol{\sigma} \parallel \mathbf{M}_\perp)$
   - *simple, powerful, but not complete*

1. **Spherical polarization analysis**

Spherical polarimetry

- establish the full relationship between the incident and the scattered beam polarizations $\boldsymbol{P}_\mathrm{i}$ and $\boldsymbol{P}_\mathrm{f}$

$$\vec{M}_\perp(\vec{Q}) = \vec{e}_Q \times \vec{M}(\vec{Q}) \times \vec{e}_Q$$



$$\vec{M}(\vec{Q}) = \sum_j f_j(\vec{Q})\vec{m}_j \exp(i\vec{Q}\vec{r}_j)\exp(-W_j)$$

Avoid averaging due to precessions around the quantization axis!

Polarized scattering cross-section

Full cross-section:

$$I = N^{+}N + \vec{M}_{\perp}^{+}\vec{M}_{\perp} + \vec{P}\left[N\left(\vec{M}_{\perp}^{+} + \vec{M}_{\perp}\right) + i\left(\vec{M}_{\perp}^{+} \times \vec{M}_{\perp}\right)\right] =$$

$$= N^{+}N + \vec{M}_{\perp}^{+}\vec{M}_{\perp} + 2P_{x}\,\mathrm{Im}\left(M_{\perp y}^{+}M_{\perp z}\right) + 2P_{y}\,\mathrm{Re}\left(NM_{\perp y}\right) + 2P_{z}\,\mathrm{Re}\left(NM_{\perp z}\right)$$

Beam polarization:

$$\vec{P}'I = \vec{P}\,N^{+}N - \vec{P}\left(\vec{M}_{\perp}^{+}\vec{M}_{\perp}\right)$$

$$+ \left(\vec{P}\,\vec{M}_{\perp}^{+}\right)\vec{M}_{\perp} + \left(\vec{P}\,\vec{M}_{\perp}\right)\vec{M}_{\perp}^{+} + iN\left(\vec{P} \times \vec{M}_{\perp}^{+}\right) - iN^{+}\left(\vec{P} \times \vec{M}_{\perp}\right)$$

$$+ N\vec{M}_{\perp}^{+} + N^{+}\vec{M}_{\perp} + i\left(\vec{M}_{\perp}^{+} \times \vec{M}_{\perp}\right)$$

Polarization transfer: matrix form

$$\vec{P}' = \mathbf{R}\vec{P} + \vec{P}''$$

final — incident polarization — created

Polarization tensor:

$$\mathbf{R}I = \begin{Bmatrix} N^2 - M_\perp^2 & -2\,\mathrm{Im}\left(N\vec{M}^+_{\perp z}\right) & 2\,\mathrm{Im}\left(N\vec{M}^+_{\perp y}\right) \\ 2\,\mathrm{Im}\left(N\vec{M}^+_{\perp z}\right) & N^2 - M_\perp^2 + 2\,\mathrm{Re}\left(\vec{M}^+_{\perp y}\vec{M}_{\perp y}\right) & 2\,\mathrm{Re}\left(\vec{M}^+_{\perp y}\vec{M}_{\perp z}\right) \\ -2\,\mathrm{Im}\left(N\vec{M}^+_{\perp y}\right) & 2\,\mathrm{Re}\left(\vec{M}^+_{\perp y}\vec{M}_{\perp z}\right) & N^2 - M_\perp^2 + 2\,\mathrm{Re}\left(\vec{M}^+_{\perp z}\vec{M}_{\perp z}\right) \end{Bmatrix}$$

Beam polarization:

$$\vec{P}''I = \begin{Bmatrix} 2\,\mathrm{Im}\left(\vec{M}^+_{\perp y}\vec{M}_{\perp z}\right) \\ 2\,\mathrm{Re}\left(N\vec{M}^+_{\perp y}\right) \\ 2\,\mathrm{Re}\left(N\vec{M}^+_{\perp z}\right) \end{Bmatrix}$$

Spherical polarimetry strategy

- determine the 9 components of the polarization transfer matrix **R** + 3 components of ***P***''

$$\vec{P}' = \mathbf{R}\vec{P} + \vec{P}''$$

final — incident polarization — created

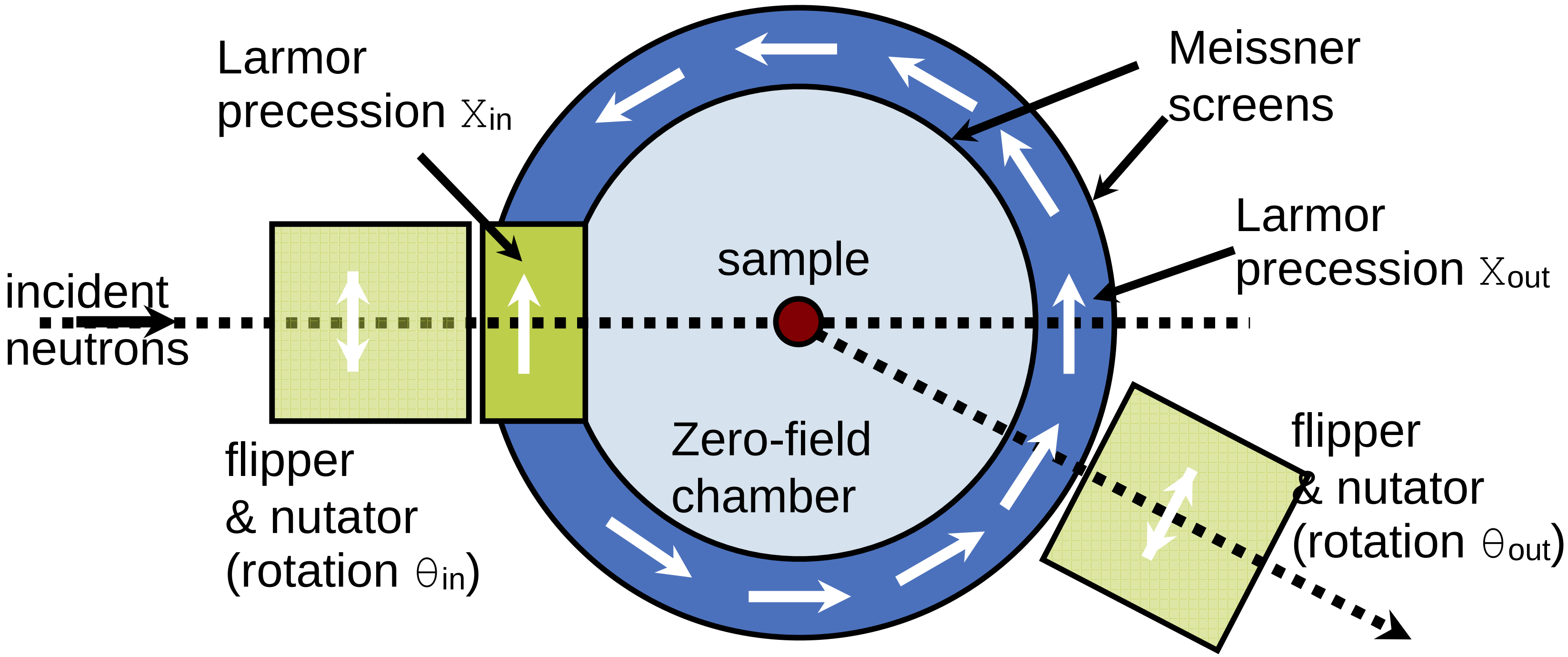
$$P'_{i,j} = \frac{I^{+}_{i,j} - I^{-}_{i,j}}{I^{+}_{i,j} + I^{-}_{i,j}}$$

Including background correction

- need field free region around the sample to avoid precession
- need to manipulate neutron spin in 3D outside the sample area

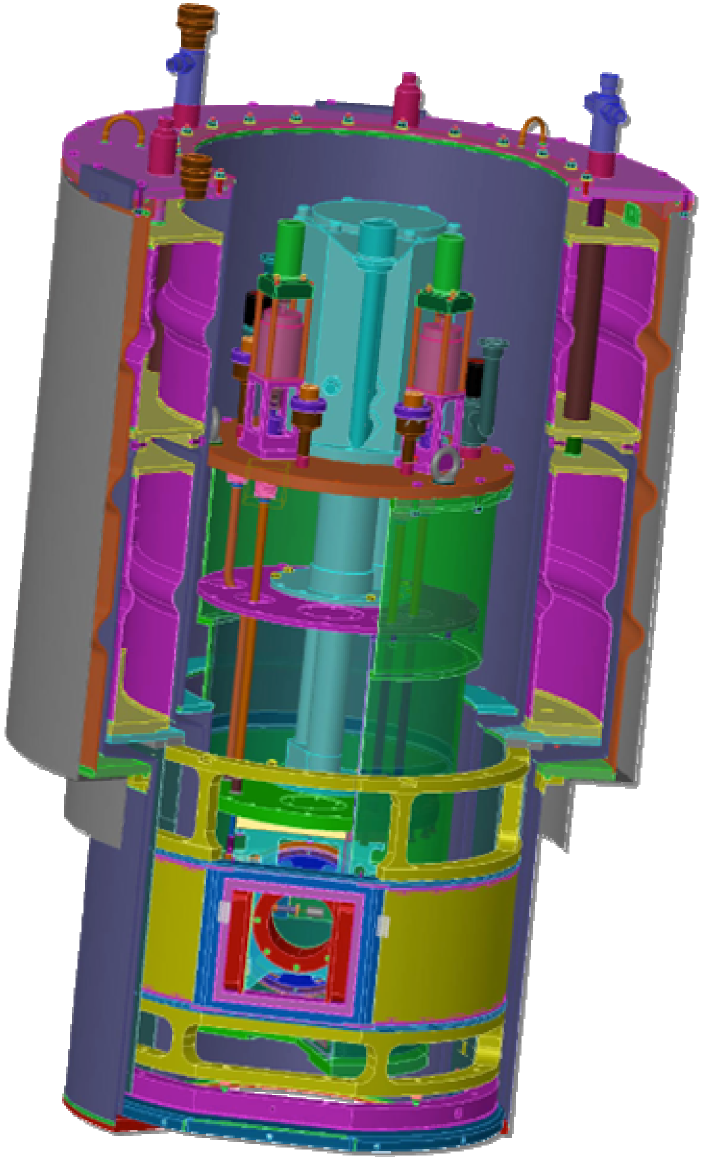
CRYOPAD scheme

CRYOPAD II



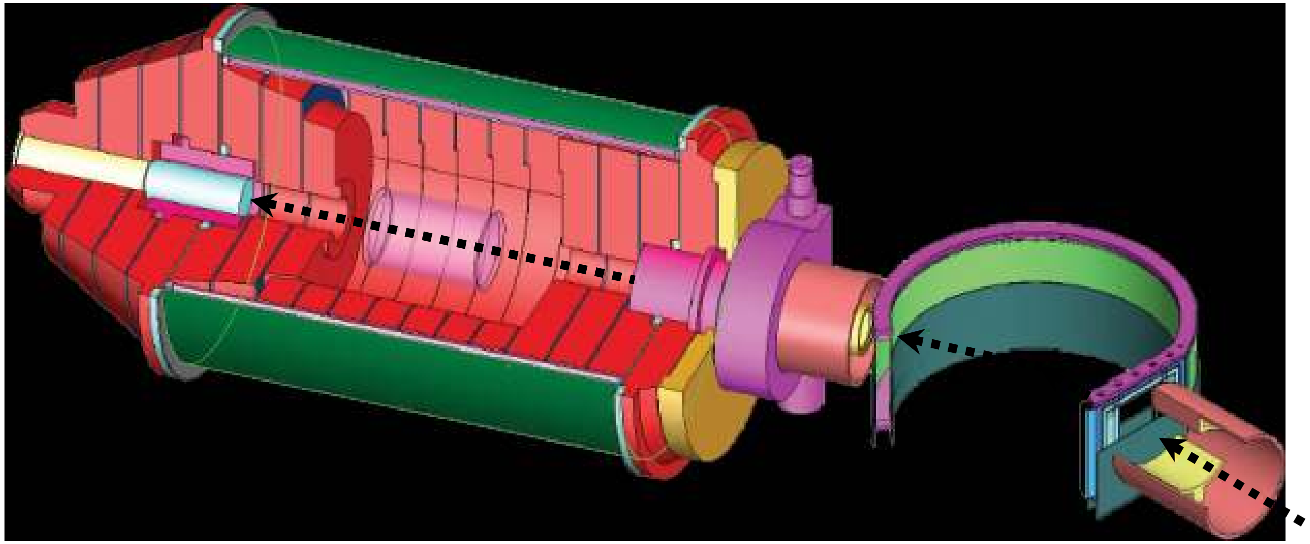
F. Tasset et al., Physica B 267}268 (1999) 69

CRYOPAD design

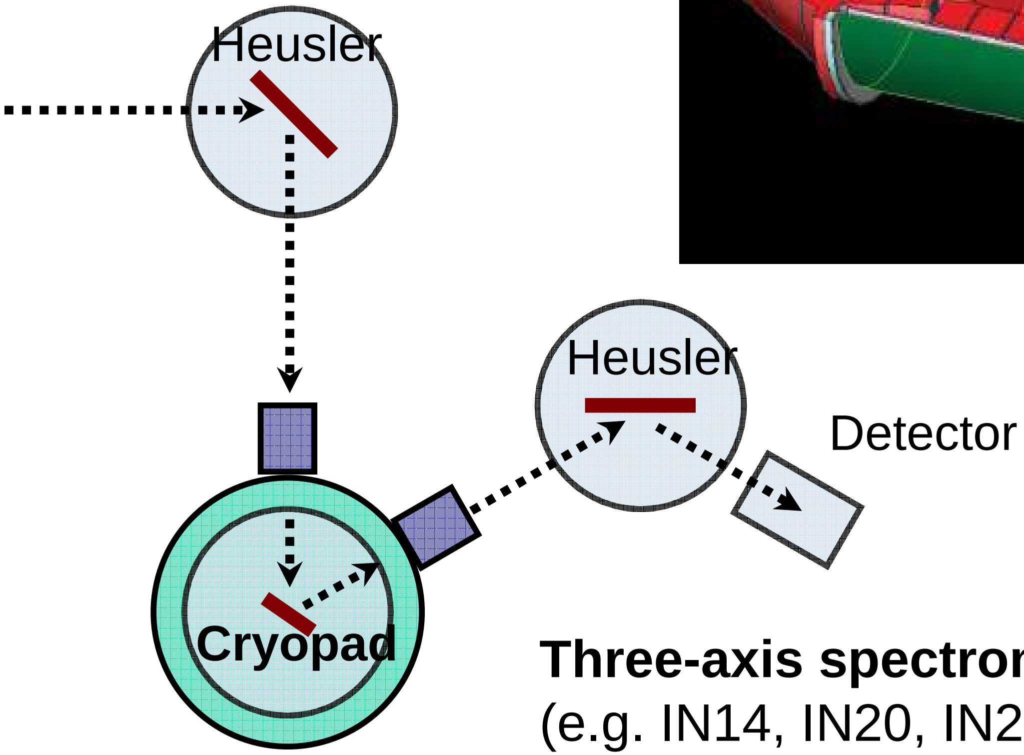


Host instruments

• provide incident and scattered beam polarization and analysis



Diffractometers with PA
(e.g. D3/Decpol @ ILL



Three-axis spectrometers
(e.g. IN14, IN20, IN22 @ ILL)

CRYOPAD on IN22

CRYOPAD & DecPol on D3

Polarization transfer: matrix form

$$\vec{P}' = \mathbf{R}\vec{P} + \vec{P}''$$

final incident created
polarization

Polarization tensor:

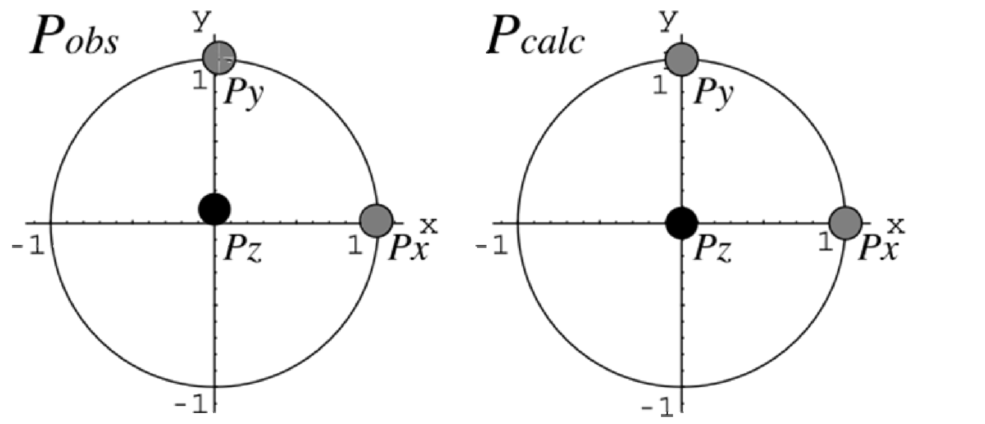
$$\mathbf{R} I = \begin{Bmatrix} N^2 - M_\perp^2 & -2\,\mathrm{Im}\left(N\vec{M}_{\perp z}^{+}\right) & 2\,\mathrm{Im}\left(N\vec{M}_{\perp y}^{+}\right) \\ 2\,\mathrm{Im}\left(N\vec{M}_{\perp z}^{+}\right) & N^2 - M_\perp^2 + 2\,\mathrm{Re}\left(\vec{M}_{\perp y}^{+}\vec{M}_{\perp y}\right) & 2\,\mathrm{Re}\left(\vec{M}_{\perp y}^{+}\vec{M}_{\perp z}\right) \\ -2\,\mathrm{Im}\left(N\vec{M}_{\perp y}^{+}\right) & 2\,\mathrm{Re}\left(\vec{M}_{\perp y}^{+}\vec{M}_{\perp z}\right) & N^2 - M_\perp^2 + 2\,\mathrm{Re}\left(\vec{M}_{\perp z}^{+}\vec{M}_{\perp z}\right) \end{Bmatrix}$$

Beam polarization:

$$\vec{P}'' I = \begin{Bmatrix} 2\,\mathrm{Im}\left(\vec{M}_{\perp y}^{+}\vec{M}_{\perp z}\right) \\ 2\,\mathrm{Re}\left(N\vec{M}_{\perp y}^{+}\right) \\ 2\,\mathrm{Re}\left(N\vec{M}_{\perp z}^{+}\right) \end{Bmatrix}$$

Spherical polarimetry

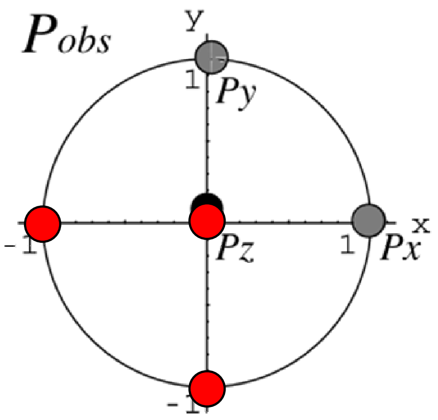
hkl	Zone axis
(004)	$\langle \bar{1}10 \rangle$



- pure nuclear scattering

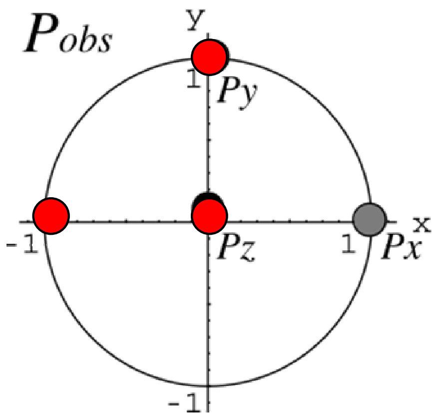
$$\mathbf{P} = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

- collinear antiferromagnet $\boldsymbol{M}_\perp \parallel \boldsymbol{z}$



$$\mathbf{P} = \begin{Bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

$$M_{\perp x} = M_{\perp y} = M_\perp / \sqrt{2}$$



$$\mathbf{P} = \begin{Bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{Bmatrix}$$

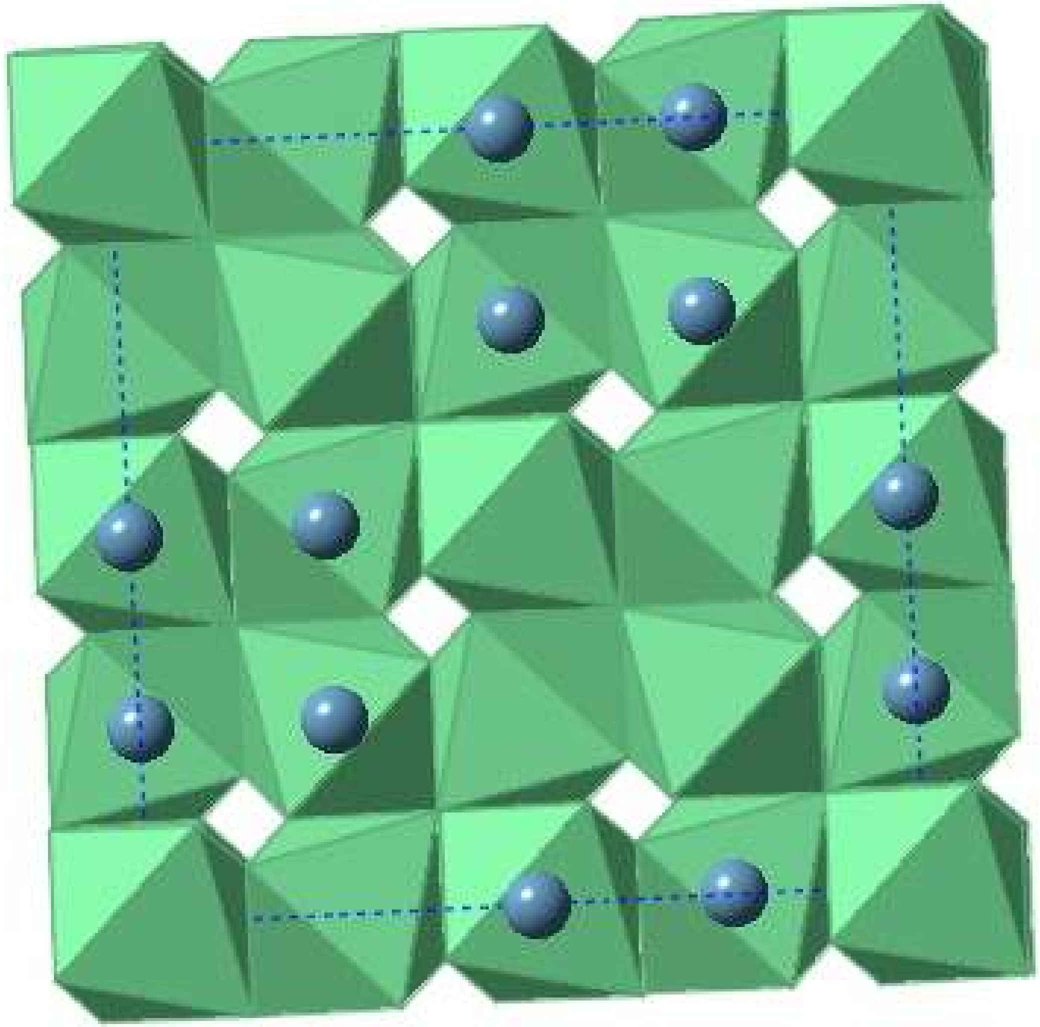
Spherical polarimetry: $Er_2Ti_2O_7$

$Er_2Ti_2O_7$

- XY <111> pyrochlore antiferromagnet

- *T_N = 1.173 K magnetic order with k = (000)*

- Er^{3+} spins perpendicular to 111 local axes



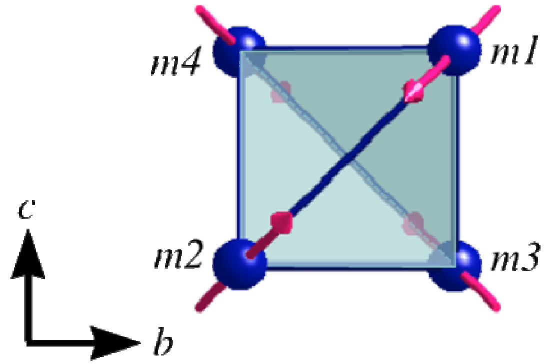
A Poole, A S Wills, E Lelievre-Berna., J. Phys.: Condens. Matter 19 (2007) 452201

Spherical polarimetry: $Er_2Ti_2O_7$

Γ_5

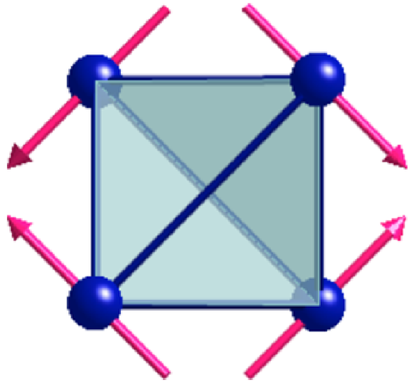
ψ_2

2	–1	–1
2	1	1
–2	–1	1
–2	1	–1



ψ_3

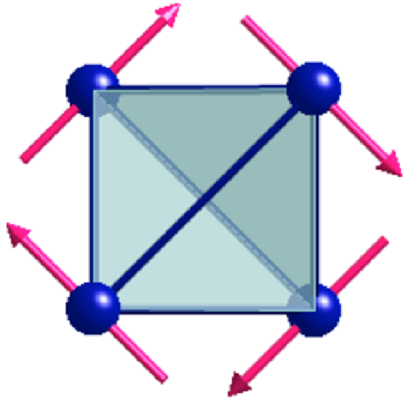
0	1	–1
0	–1	1
0	1	1
0	–1	–1



Γ_7

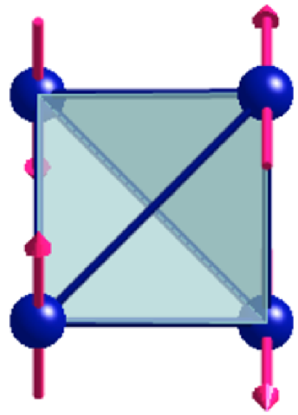
ψ_4

0	1	–1
0	–1	1
0	–1	–1
0	1	1



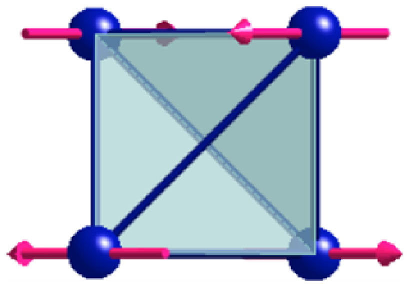
ψ_5

–1	0	1
1	0	1
1	0	–1
–1	0	–1



ψ_6

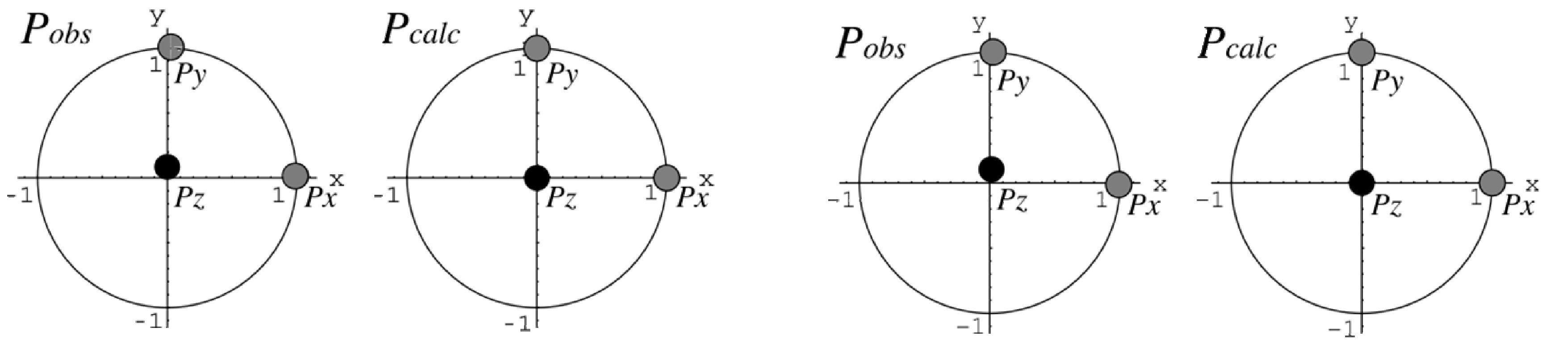
1	1	0
–1	1	0
1	–1	0
–1	–1	0



NEUTRONS FOR SCIENCE

Spherical polarimetry: $Er_2Ti_2O_7$

hkl	Zone axis	*hkl*	Zone axis
(004)	$\langle\bar{1}10\rangle$	(400)	$\langle 001\rangle$

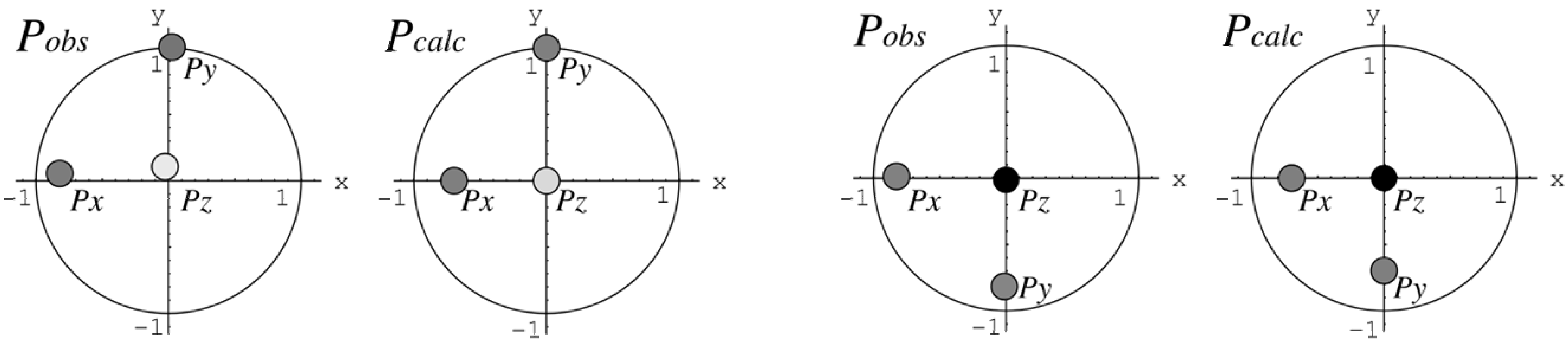


- $M(004) \approx M(400) \approx (m_1+m_2+m_3+m_4) \approx 0$
- purely antiferromagnetic structure

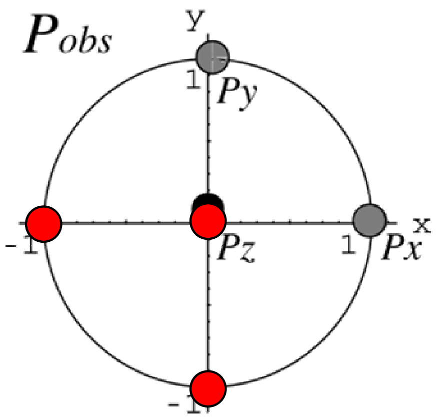
Spherical polarimetry: $Er_2Ti_2O_7$

hkl	Zone axis
(220)	$\langle\bar{1}10\rangle$

hkl	Zone axis
(220)	$\langle 001\rangle$



- collinear antiferromagnet $\boldsymbol{M}_\perp \parallel \boldsymbol{z}$

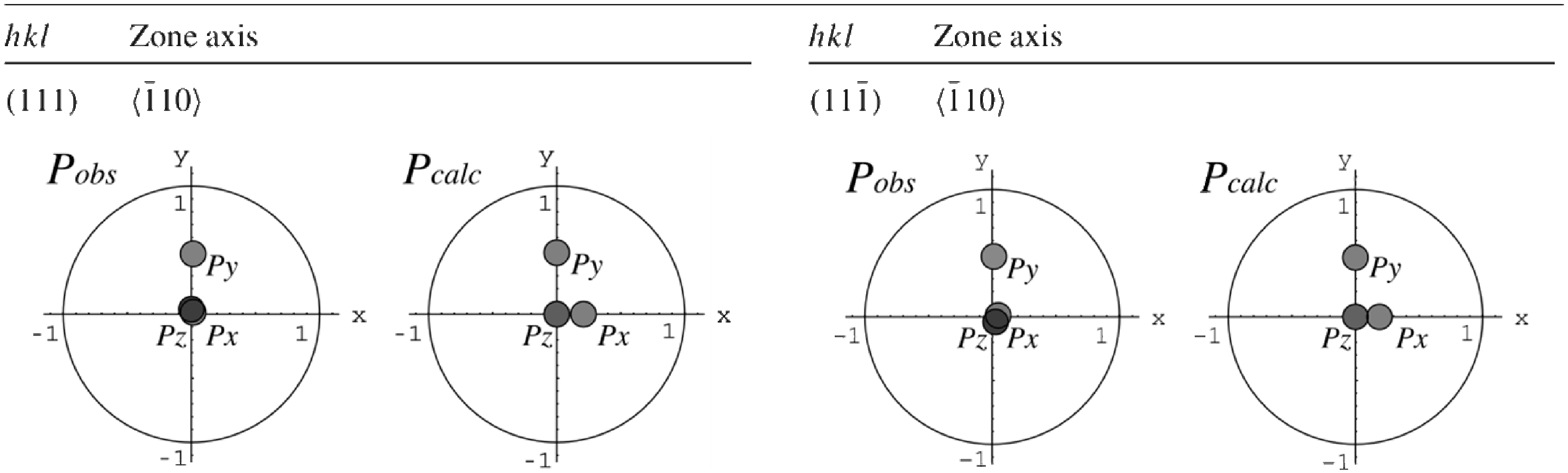


$$\mathbf{P} = \begin{Bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

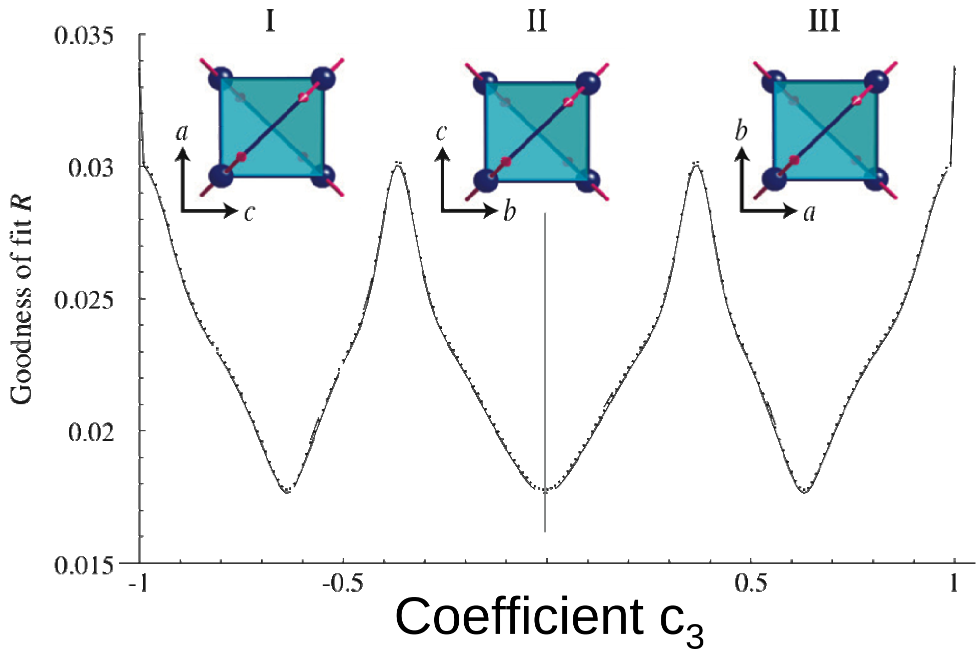
$M_\perp$ || <001>

Only provided by m_2, m_3 in basis vectors of Γ_5

Spherical polarimetry: $Er_2Ti_2O_7$



Strong depolarization:
need modelling of domain effects

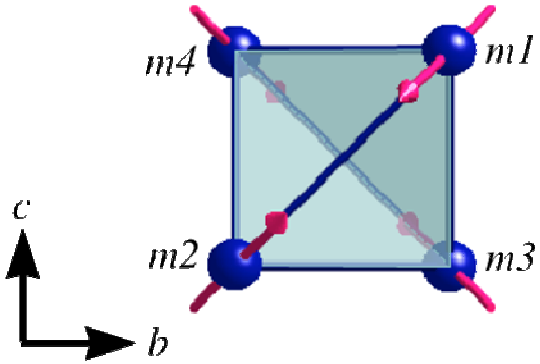


Spherical polarimetry: $Er_2Ti_2O_7$

Γ_5

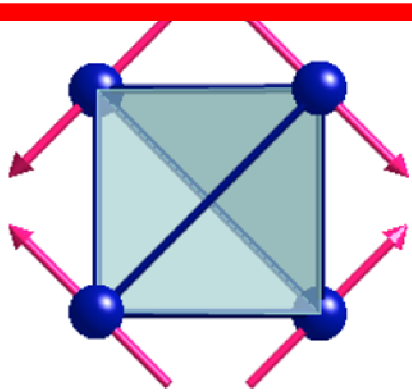
ψ_2

2	−1	−1
2	1	1
−2	−1	1
−2	1	−1



ψ_3

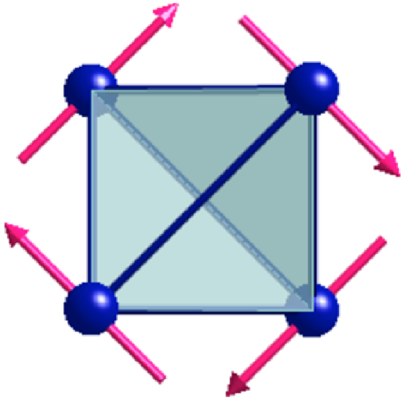
0	1	−1
0	−1	1
0	1	1
0	−1	−1



Γ_7

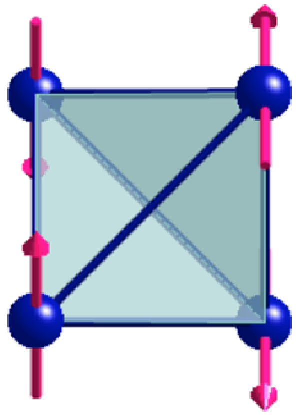
ψ_4

0	1	−1
0	−1	1
0	−1	−1
0	1	1



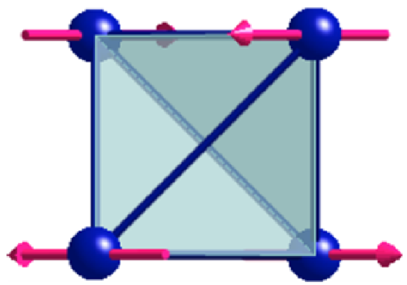
ψ_5

−1	0	1
1	0	1
1	0	−1
−1	0	−1



ψ_6

1	1	0
−1	1	0
1	−1	0
−1	−1	0



Neutron polarization analysis ...

1. **Fundamental equations**
   - *only* $\mathbf{M} \perp \mathbf{Q}$ *gives rise to scattering*
   - *beam polarization depends on* $(\boldsymbol{\sigma} \cdot \mathbf{M}_{\perp})$

1. **Longitudinal (diagonal) polarization analysis**
   - *beam polarization by net magnetization* $\mathbf{M}_{\perp}$ (linear or chiral)
   - *spin-flip* $(\boldsymbol{\sigma} \perp \mathbf{M}_{\perp})$ *and non-spin-flip scattering* $(\boldsymbol{\sigma} \parallel \mathbf{M}_{\perp})$
   - *simple, powerful, but not complete*

1. **Spherical polarization analysis**
   - *complete and powerful technique*
   - *Complicated by domain averaging*

ZERO FIELD: *invaluable for work in superconducting state!*

Atomic scattering amplitudes

$$U^{++} = b - pS_{\perp z} + B\vec{I}_z$$

$$U^{--} = b + pS_{\perp z} - B\vec{I}_z$$

$$U^{+-} = -p\left(S_{\perp x} + iS_{\perp y}\right) + B\left(\vec{I}_x + i\vec{I}_y\right)$$

$$U^{-+} = -p\left(S_{\perp x} - iS_{\perp y}\right) + B\left(\vec{I}_x - i\vec{I}_y\right)$$

- magnetic scattering amplitude

$$p = \frac{\gamma e^2}{2mc^2}\, gS\, f(Q)$$

Scattering cross-sections

Contribution	Elastic scattering	Inelastic scattering
(n) Nuclear	$\sigma_n = NN^*$ $\{\vec{P}_f\sigma\}_n = \vec{P}_i\ \sigma_n$	$\sigma_n = \frac{k_f}{k_i}\mathcal{H}\left(N_{-\vec{Q}}, N_{\vec{Q}}\right)$ $\{\vec{P}_f\sigma\}_n = \vec{P}_i\ \sigma_n$
(m) Magnetic (I)	$\sigma_m = \vec{M}_\perp \cdot \vec{M}_\perp^*$ $\{\vec{P}_f\sigma\}_m = -\ \vec{P}_i\ \sigma_m\ + \ldots$ $\ldots\ 2\Re\left(\vec{M}_\perp\left(\vec{P}_i \cdot \vec{M}_\perp^*\right)\right)$	$\sigma_m = \frac{k_f}{k_i}\ S_{\alpha\beta}\,\delta_{\alpha\beta}$ $\{\vec{P}_{f,\alpha}\sigma\}_m = \frac{k_f}{k_i}P_{i\beta}\cdot\ldots$ $\ldots\ [(S_{\alpha\beta} + S_{\beta\alpha}) - \delta_{\alpha\beta}\delta_{\beta\alpha}]$ $S_{\alpha\beta} = \mathcal{H}\left(M^\alpha_{\perp,-\vec{Q}}, M^\beta_{\perp,\vec{Q}}\right)$
(c) Magnetic (II)	$\sigma_c = \imath\vec{P}_i \cdot \left(\vec{M}_\perp^* \wedge \vec{M}_\perp\right)$ $\{\vec{P}_f\sigma\}_c = -\ \imath\left(\vec{M}_\perp^* \wedge \vec{M}_\perp\right)$	$\sigma_c = \frac{k_f}{k_i}\ \imath\, S_{\alpha\beta}\,\epsilon_{\alpha\beta\gamma}\ P_{i\gamma}$ $\{\vec{P}_{f,\alpha}\sigma\}_c = -\frac{k_f}{k_i}\imath\epsilon_{\alpha\beta\gamma}S_{\beta\gamma}$ $S_{\alpha\beta} = \mathcal{H}\left(M^\alpha_{\perp,-\vec{Q}}, M^\beta_{\perp,\vec{Q}}\right)$
(i) Nuclear–magnetic	$\sigma_i = 2\vec{P}_i \cdot \Re\left(N^*\vec{M}_\perp\right)$ $\{\vec{P}_f\sigma\}_i = 2\Re\left(N^*\vec{M}_\perp\right)\ +$ $2\vec{P}_i \wedge \Im\left(N^*\vec{M}_\perp\right)$	$\sigma_i = \frac{k_f}{k_i}\ \imath\vec{S}_+ \cdot \vec{P}_i$ $\{\vec{P}_f\sigma\}_i = \frac{k_f}{k_i}\left(\vec{S}_+ + \imath\vec{S}_- \wedge \vec{P}_i\right)$ $\vec{S}_\pm = \mathcal{H}_\pm\left(N_{-\vec{Q}}, \vec{M}_{\perp,\vec{Q}}\right)$